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IX. *Note on the Calculation of the Coefficient of Diffusion of a Salt at a Definite Concentration.* By A. GRIFFITHS, *D.Sc.*

RECEIVED OCTOBER 20, 1916. REVISED JANUARY 26, 1917.

IN the calculation of the coefficient of diffusion, by B. W. Clack, a simple relation is assumed between the density of a solution of a salt and the concentration. This simple relation is only approximately correct and compromises are made which require justification.

This note suggests—

(1) A method of calculating the coefficient of diffusion which, to a high degree of theoretical accuracy, gives values for the coefficient which are independent of a precise relationship between density and concentration ; and

(2) Justifies the method of calculation adopted by B. W. Clack.

§ 1. Consider the case of a tube of unit cross-section and of length L_N. The lower end dips into a closed vessel which contains a solution whose concentration is N grams per cubic centimetre. The upper end opens into a large unclosed vessel containing water.

Let v_0=velocity of water at upper end of tube.

n=concentration at distance l from top of tube.

ρ_n=density of solution at distance l from top of tube.

c_N=mass of salt transmitted per second after attainment of the steady state when concentration at bottom of tube is N.

$f(n)$=coefficient of diffusion at concentration n.

δ_N=ratio of increment of volume of solution to mass of salt added at concentration N.

Then

$$\frac{v_0 n}{\rho_n - n} + f(n)\frac{dn}{dl} = c_N.$$

$$v_0 = c_N \delta_N,$$

whence

$$\int_0^N \frac{f(n)(\rho_n - n)}{\rho_n - n + n\delta_N} dn = \int_0^{L_N} c_N dl = C_N L_N.$$

Differentiating both sides of the equation with respect to N

$$f(N)\left\{\frac{\rho_N - N}{\rho_N + N + N\delta_N} - \frac{d\delta_N'}{dN}\int_0^N \frac{f(n)}{f(N)} \frac{n(\rho_n - n)}{(\rho_n - n + n\delta_N)^2} dn\right\}$$

$$= \frac{d}{dN}(L_N C_N) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The second term within the bracket ({ }) is relatively small; it is approximately correct to neglect it and to write

$$f(N)=\frac{d(L_N C_N)}{dN}\times\frac{(\rho_N-N+N\delta_N)}{(\rho_N-N)} \quad . \quad . \quad . \quad (2)$$

Thus, if the relation between $L_N C_N$ and N be determined experimentally, the approximate value of the coefficient of diffusion at concentration N is given without any hypothesis as to the precise relationship between the density and the concentration.

Having found from equation (2) a first approximation to $f(N)$, and therefore to $f(n)$, (for the forms of the two functions are identical), the value of the second term within the brackets ({ }) can be determined; and, finally, equation (1) can be used to determine $f(N)$ with an increased and high degree of accuracy.

Let us consider the case of a solution which has the properties of an aqueous solution of KCl, but in which the relationship between the density and the concentration may be specified by

$$\rho_N=1+0{\cdot}62N-0{\cdot}13N^2.$$

This equation is approximately true for a solution of potassium chloride at 18°C. In the published work of B. W. Clack and of the author a linear relationship is assumed.

This equation gives

$$\delta_N=\frac{0{\cdot}38+0{\cdot}26N}{1+0{\cdot}13N^2}$$

and

$$\frac{d\delta_N}{dN}=\frac{0{\cdot}26-0{\cdot}0988N-0{\cdot}0338N^2}{(1+0{\cdot}13N^2)^2}.$$

The values of $f(N)$ deduced by intrapolation and extrapolation from B. W. Clack's Paper are as follows :—

$$f(0{\cdot}05)=1{\cdot}545\times10^{-5}\,;\quad f(0{\cdot}1)=1{\cdot}651\times10^{-5}$$
$$f(0{\cdot}15)=1{\cdot}757\times10^{-5}\,;\quad f(0{\cdot}20)=1{\cdot}863\times10^{-5}$$

The value of the integral in the second term within the brackets ({ }) may be found by plotting a curve on squared paper and when $N=0{\cdot}2$ (1) becomes, after a little re-arrangement,

$$f(N)\frac{\rho_N-N}{\rho_N-N+N\delta_N}\left\{1-0{\cdot}0050\right\}=\frac{dL_N C_N}{dN}. \quad . \quad . \quad (3)$$

[Note.—$N=0{\cdot}2$.]

It may be particularly noted that equation (3) holds only for the special case that $N=0{\cdot}2$. A different value of N would

necessitate a calculation of the new value of the second term within the brackets ({ }).

It may incidentally be mentioned that an approximate value of the integral within the brackets ({ }) of equation (1) is $N^2/2$. If this approximation be used then the second term within the brackets ({ }) of equation (2) becomes 0·0052.

§ 2. B. W. Clack in his Papers, writes :—

$$C_N L_N = {}_0K_N\left(1-\frac{N\delta_N}{2}\right)N.$$

$$\therefore \frac{dC_N L_N}{dN} = N\left(1-\frac{N\delta_N}{2}\right)\frac{d}{dN}({}_0K_N) + {}_0K_N\left(1-N\delta_N-\frac{N^2}{2}\frac{d\delta_N}{dN}\right)$$

and, when $N=0{\cdot}2$, from equation (3),

$$f(N)=\frac{(\rho_N-N+N\delta_N)\left[{}_0K_N\left(1-N\delta_N-\frac{N^2}{2}\frac{d\delta_N}{dN}\right)+N\left(1-\frac{N\delta_N}{2}\right)\frac{d}{dN}({}_0K_N)\right]}{(\rho_N-N)\{1-0{\cdot}0050\}}$$

$$=0{\cdot}9998\,{}_0K_N+0{\cdot}2104\frac{d}{dN}({}_0K_N) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

[Note.—$N=0{\cdot}2$.]

The equation adopted by B. W. Clack in his recent Paper is

$$f(N)={}_0K_N+\frac{\left(1-\frac{N\delta_N}{2}\right)N}{1-N\rho_N}\frac{d_0K_N}{dN},$$

whence

$$f(N)={}_0K_N+0{\cdot}2093\frac{d}{dN}({}_0K_N). \quad . \quad . \quad . \quad . \quad . \quad (5)$$

[Note.—$N=0{\cdot}2$.]

To compare the numerical values given by (4) and (5), let ${}_0K_N=1{\cdot}7\times10^{-5}$, when $N=0{\cdot}2$. Also let $\frac{d}{dN}({}_0K_N)=0{\cdot}5\times10^{-5}$.

Equation (4) gives $f(N)=1{\cdot}8049\times10^{-5}$; equation (5) gives $f(N)=1{\cdot}8047\times10^{-5}$.

Thus, the equation used by B. W. Clack is completely justified.

It may be mentioned that the simple equation given by B. W. Clack in an earlier Paper $\left(\text{viz., } f(N)={}_0K_N+N\frac{d}{dN}({}_0K_N)\right)$ gives a good approximation to the correct value.

ABSTRACT.

In the calculation of the coefficient of diffusion, by B. W. Clack, a simple relation is assumed between the density of a solution of a salt and the concentration. This simple relation is only approximately correct, and compromises are made which require justification.

This note—

1. Suggests a method of calculating the coefficient of diffusion which, to a high degree of theoretical accuracy, gives values for the coefficient which are independent of a precise relationship between density and concentration ; and

2. Justifies the method of calculation adopted by B. W. Clack.

DISCUSSION.

Mr. B. W. Clack : The coefficient of diffusion is not a constant, but varies with the concentration to a considerable extent. In most experimental work the coefficient varies at different positions in the apparatus, and any work which helps towards the proper interpretation of results is of value.

X. *A Special Test on the Temperature Effect of Gravitation.**

By P. E. SHAW, *B.A., D.Sc., and* C. HAYES, *B.Sc.*

RECEIVED DECEMBER 29, 1916

1. *Introduction.*

IN the "Philosophical Transactions" of the Royal Society for 1916 † there is a Paper by one of us dealing with the possible existence of a temperature coefficient in the law of gravi-

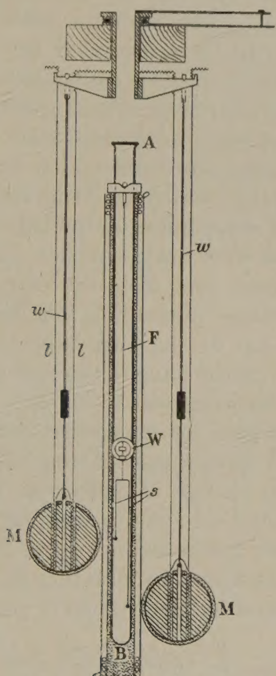


FIG. 1.—AN ELEVATION OF THE APPARATUS.

The central vacuum tube contains the torsion balance whose movements are observed through the window W. The special brass cylinders on which the contacts are made are shown thick on the hanging wires *w w*.

tation, and giving an account of experiments made to discover this coefficient.

The essential parts of the apparatus used are shown in Fig. 1.

* Briefly described in the discussion on Gravitation at a meeting of Section A, British Association, Newcastle, 1916.

† Shaw, Vol. CCXVI., pp. 349-392.

From this sketch many details have been omitted, but full drawings and description will be found in the Paper quoted.

The glass tube A B is 120 cm. long and 5 cm. bore, and is exhausted to a pressure of 15 mm. of mercury. Two fine wires, *ss*, of length 15 cm. and 35 cm. respectively, are suspended from the ends of a torsion beam which is hung by a quartz fibre, F, 60 cm. in length. Each of these wires carries a very pure silver ball weighing 2·5 gm. The movements of the torsion beam, carrying a mirror which reflects light from a distant scale, are observed through the window W by a telescope 4·5 metres distant. The vacuum tube is well lagged with cotton wool, and this is surrounded by a helix of "compo" tubing through which water is continually passing.

Outside the vacuum tube a lead sphere, weighing 47 kilos, was placed on the same horizontal level as each of the silver balls. These spheres were suspended by stout copper wires, *w w*, from a turntable which could be rotated about a vertical axis with ease and smoothness on the ball-bearings shown in the figure Two holes were drilled in each sphere to admit a heating coil of nichrome, and over the cotton-wool covering the spheres there is a layer of tinfoil. The leads *l l* for the heating coils are carried up to the turn-table arms, and, therefore, exert no influence on the hang of the spheres. Stops are placed on the dial of the swing-table in such positions (found by experiment) as to give the maximum deflection of the torsion system as the lead spheres moved from the A to the B positions (*see* Fig. 2). This change of deflection, called the range, was measured when the lead spheres were at 18°C., and also when they were at other temperatures up to 250°C. The temperature coefficient was found by dividing the change of range by the whole range and by the change of temperature.

2. *Object of the Test.*

From these experiments it was deduced that, at least for the materials used, there is a temperature effect of gravitation when the emperature of the large mass rises, while that of the small mass remains constant.

Supposing that this effect is linear it can be expressed by the relation

$$F = G(1+\alpha\theta)\frac{Mm}{d^2},$$

where F is the force of attraction and θ is the increment in temperature of the large mass, G being the Newtonian con-

stant at ordinary temperatures, and d being the distance between the centre of mass of either large sphere and its corresponding small mass. The value found for α was $+1{\cdot}2 \times 10^{-5}$ per 1°C.

There are many possible sources of error mentioned in the Paper which might account for the observed small change in

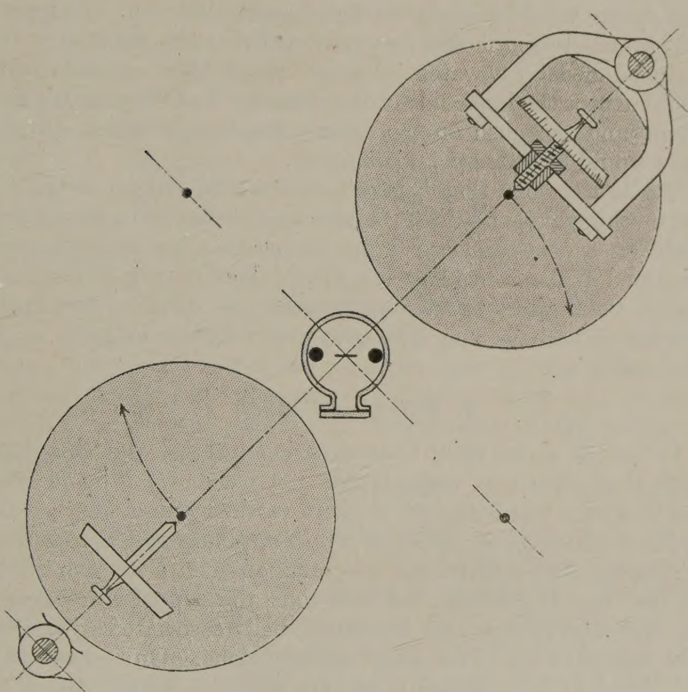


FIG. 2.—A PLAN VIEW.

The large spheres are shown with the hanging wires in section at their centres. The section of the vacuum tube is seen at the centre. The micrometer and frame are shown on the right and the micrometer alone on the left for the A position. The B position is indicated by a section of the hanging wires.

the deflection. All these were considered to have been satisfactorily eliminated by the precautions taken; but another possible source was mentioned in the criticism after the reading of the Paper. This will be found on p. 391 as follows: "It is suggested that with the high temperature of the lead spheres (250°C.) considerable convection currents would be set up

round them even when lagged." In the region where the spheres are close to the tube, the air velocity might be very much larger than on the outer regions ; and, as a consequence, a difference of pressure would be called into play, pushing the suspended spheres towards the tube, and therefore increasing the actual attraction on the suspended small balls. It is calculated that an *inward* displacement of 0·15 mm. in each lead sphere would account for the change observed. In reply to this criticism it should be mentioned that the vacuum tube is surrounded by a water screen at about 11°C., so that one would expect the inner side of the sphere to be colder than elsewhere, and the push on the sphere due to convection differences would be *outwards*, not *inwards*.

It was thus felt that this criticism would not be weighty, but we thought that a direct observation on the point should be made on the apparatus exactly as used in the experiments described in the above Paper. Again, there may be disturbing effects other than convection tending to displace the lead spheres radially. The proposed test should indicate any such influence.

3. *Method of Testing.*

To put the above point to an experimental test, the following method of procedure was adopted. A piece of T-iron, to one end of which was attached a short vertical rod of steel, was fastened to each of the four wooden pillars supporting the apparatus. The attachment was made in such a manner that for the A or B position, the steel rods, the copper wire carrying the lead sphere and the silver ball (associated with the lead sphere) were in the same vertical plane. On to the steel rod fitted an arrangement, shown in Fig. 2, which carries a micrometer. In use this micrometer screw was adjusted by swinging the micrometer on its vertical pin and by turning the screw until it just made contact with the vertical wire. When this occurred an electric circuit was completed through the wire and the micrometer, which caused a sound in a telephone receiver, and the reading of the micrometer was then taken. The position of the copper wire was thus accurately determined, and any movement to or from the silver sphere could be detected. Unless the copper wire were quite straight, we should not be able to repeat our readings if any rotation occurred about a vertical axis passing through the wire.

In order to overcome this difficulty, brass cylinders, each 8 cm. long, were accurately turned, so that the inside and outside surfaces were concentric and the inner diameter equal to that of the copper wire. A small longitudinal opening was made in each cylinder, through which the wire was passed, and attached with a little solder. Any slight rotation of the lead spheres will now introduce no error, seeing that the axis of rotation is the geometric axis of the outside surface.

The method of taking the readings was as follows: The micrometer readings for the two wires were taken for the A position and then for the B position (Part I. in table). The temperature of the lead spheres was then raised to 220°C., and the four micrometer readings again taken (Part II. in table). This was again repeated when the spheres were cold (Part III. in table). In order to be certain that the torsion system was moving freely the scale reading of the torsion beam was taken roughly for each of the above positions.

To assist in keeping the position of the steel rods fixed during the experiment the four pieces of T-iron were wrapped with felt, cotton and cardboard, and the four wooden pillars were bound together by other pieces of T-iron, which were similarly lagged to prevent appreciable rise of temperature during the experiment.

4. *Readings.*

The readings are shown in the table on next page. In the first column is the position of the lead balls, while in the second is the tube reading. The latter is obtained by the mirror which is attached to the vacuum tube, which records any rotation of the apparatus as a whole. In the third column are the readings of the position of the torsion system. This was never at rest, but from two consecutive readings of the extreme positions the exact position of rest can be calculated if the decrement of the system be known. In the next column we have the temperature of the lead balls.

A wooden pillar is referred to as the north, south, east or west pillar, according to its position with respect to the cardinal points of the compass. Thus in the A position of the lead balls we have readings of the micrometer from the east and west pillars, while for the B position they are from the north and south pillars. Since a movement of the wire towards the vacuum tube will give a *smaller* micrometer reading, it will be seen that the *sum* of the east and west (or north and south)

Table of Results.

	Pos.	T.R. mm.	R. mm.	θ_L.	Radial readings of M.M. in mm.					
					N.	S.	E.	W.	N+S.	E+W.
		...	237·6	...	6·516	...	...	...	...	...
I.	B	324·9	382·7	19°C.	6·525	15·841	...	...	...	...
	A	324·3	220·4	...	...	...	12·350	10·310	...	...
			64·9	19°C.	...	...	12·360	10·303	...	...
	B	...	...	19°C.	6·527	15·845	...	...	...	...
					6·523	15·843	12·355	10·307	22·366	22·662
II.	A	324·0	53·9	216°C.	...	...	...	...	...	...
			173·9	202°C.	...	...	12·385	10·284	...	...
	B	...	...	206°C.	6·540	15·862	...	...	...	...
				190°C.	6·542	15·855	...	...	...	...
	A	...	...	199°C.	...	...	...	...	...	...
				182°C.	...	...	12·415	10·267	...	...
	B	324·8	404·1	190°C.	...	...	...	...	...	...
			268·9	176·C.	6·540	15·858	...	...	...	...
					6·541	15·858	12·400	10·275	22·399	22·675
III.	B	324·9	433·1	...	...	...	...	...	...	...
			249·9	23°C.	6·535	15·830	...	...	22·365	...
	A	324·4	9·7	...	...	...	...	...	...	...
			204·6	23°C.	...	...	12·340	10·327	...	22·667
Mean for cold									22·366	22·665
,, hot									22·399	22·675
Difference									+0·033	+0·010
Mean difference = +0·021 mm.										

readings, together with some unknown *constant* length will give us the distance between the wires for the A (or B) position of the lead balls. The difference in the numbers in the ninth and tenth columns are due to the chance that the vertical pins for supporting the micrometer are not equidistant from the central axis.

From these results it will be seen that for the A position we have an OUTWARD movement of 0·01 mm. and for the B position an OUTWARD movement of 0·03 mm., giving a mean outward movement of 0·02 mm.

5. *Result.*

As pointed out earlier the temperature effect observed in the original research could be produced if an INWARD movement of 0·15 mm. occurred in *each* lead sphere.

The present test shows that the total movement of *both* lead spheres is 0·02 mm., so that *each* lead sphere would only move

0·01 mm. This is one-fifteenth of the effect looked for, and it is *outward—i.e.*, in the contrary sense. Thus, the gravitation-temperature effect will now have to be increased slightly.

Before calculating the correction to be applied to the temperature coefficient for this small observed displacement, we notice that it would require a rise in temperature of only 4°C. in the turn-table beam to produce the outward movement of 0·02 mm. It is not unlikely that the beam would rise in temperature by this amount, and we feel incl ned to attribute the effect observed in this test to this cause rather than to any Bernouilli effect or radiation pressure.

In the original research the probable result is

$$\alpha = +(1{\cdot}2 \pm 0{\cdot}05) \times 10^{-5}.$$

As a result of the present test this must now be amended to—

$$\alpha = +(1{\cdot}3 \pm 0{\cdot}05) \times 10^{-5}.$$

In conclusion, we might say that the present method, rather than an optical one, was chosen chiefly because of the difficulty of working any optical device with the gravitation apparatus as already installed.

In order to avoid small errors of displacement it seems advisable always in future to apply a test for the horizontal distance between the suspending wires in making gravitation-temperature experiments, and we are inclined in future to give opti al methods a trial as the above method is rather tedious. We find that mechanical contact of the micrometer screw on the suspending wires tends to set these in vibration ; whereas if a microscope or an optical lever device were used there would, of course, be no actual contact or, in the latter case, only a very slight one.

ABSTRACT.

In the Philosophical "Transactions" of the Royal Society, Vol. CCXVI., pp. 349 to 392, there is a Paper by one of the authors dealing with the possible existence of a temperature coefficient of the constant of gravitation. It was suggested in the discussion that the effect might be due to an inward displacement of the large lead spheres, at the higher temperatures, due to convection currents.

In the present Paper experiments are described in which this point is tested by micrometric measurements of the positions of the supporting wires. It is shown that, at the higher temperatures, there is a small *outward* displacement of the spheres, probably due to the expansion of the crosshead from which they are suspended. A slightly *higher* value has, therefore, to be given to the temperature coefficient of gravitation.

DISCUSSION.

Mr. F. E. SMITH suggested that since it appeared to be necessary to measure the deflection of the large spheres in order to arrive at the correct result, this could be more advantageously done with an arrangement of microscopes than with the micrometers employed.

Prof. C. V. BOYS thought the subject was one of vital importance. He was not yet convinced that the results obtained by Dr. Shaw were necessarily true; but the apparatus was in many ways so beautifully designed that it was difficult to see what could give rise to the discrepancy. It seemed to him that Dr. Shaw's method of measuring the constant at two different temperatures was not the best way of measuring the *difference* between these values, which was thus subject to the inaccuracy of the complete determination. It seemed a better plan to have two sets of balls, one in the A position and one in the B position, and to measure any change in the deflection produced when one pair were hot and the other cold, and *vice versa*. This would be a null method, and would measure only the temperature coefficient if such existed. He had mentioned this to Dr. Shaw, who had brought forward some difficulty in connection with it which he could not at the moment remember.

Mr. C. C. PATERSON asked if it was not possible for convection currents to be set up inside the vacuum tube and to affect the small spheres.

Dr. SHAW replied as follows: Mr. Smith would prefer a set of microscopes to a set of electric micrometers for the measurements. We state at the end of the Paper that we should have tried, and very likely adopted, this method if we had been able; but the scaffolding and permanent tables surrounding the gravitation apparatus prevented that free access which would have been required for optical measurement. We should on another occasion make attempts to use a microscope, though, of course, there are special difficulties (*e.g.*, vibration) involved in its use on a long wire. The alternative method suggested by the President, ideal in some experiments, would seem to suffer in comparison with the method used in two respects. (1) The observed temperature effect would be reduced to one-half, since the hot sphere would, in effect, act on one side only, instead of acting, as at present, on both sides. The effect, already small, would thus be measurable with only half the accuracy. (2) A more serious technical trouble would arise from the complete inability to check the zero of the torsion beam. This zero may drift in the method so far used, without directly invalidating the result; but in the proposed method any drift would enter the result as a direct error. In the method used we can observe zero continually, and the constancy of the zero really affords some indication of the accurate working of the apparatus. The possibility of movements of gas in the vacuum must be kept in mind, as Mr. Paterson urges. The arguments against the occurrence of these movements in the experiments are given in the original Paper. The hot lead spheres are separated from the vacuum chamber containing the torsion system by a screen of flowing water at constant temperature, and by a composite lagging of cotton wool, flannel, &c. The only agency which could cause gas movements would be the *one* factor which changes during the experiment—viz., the heat introduced into the lead. This heat on leaving the lead would have to pass in succession through cotton wool, flowing water, cotton wool, glass tubing, silver tubing. On reaching the inner wall of the silver tubing it would be liberated to the gas in the vacuum chamber. Now, the cycle from the A position to the B position and back to the A position takes only 12 minutes. Is it conceivable that the supposed flow of heat from the lead to the gas can be so abundant and quick through the composite screen that the gas movement can reverse its direction in six minutes? But, apart from conjecture, actual tests were made of the effect when the lead spheres were kept a very short time at the A and B position. No time effect could be observed. Again, the lagging was varied greatly in thickness without observable effect. The test described in this Paper is the only one I have been able to make since 1915, on account of the practical impossibility of obtaining necessary material (copper or tungsten) at present.

XI. *To Measure the Pressure in a High Vacuum by Observation of Logarithmic Decrement.* By P. E. Shaw, *B.A., D.Sc.*

Received January 9, 1917.

In experiments on the Newtonian constant ("Phil. Trans.," May, 1916) I used a torsion balance, in a vacuum which varied in different cases from 15 mm. to 0·01 μ pressure. Before the vacuum vessel is sealed off the pressure is determined by a McLeod gauge, but after sealing off there does not seem to be any ready, accurate method in general use for this determination. But by observing the damping of the vibration of the torsion system in *high* vacuum, we have a means at our disposal. We merely use the ordinary readings of amplitude, and no special readings or special apparatus are required for the determination of pressure.

For a vacuum pressure of the order of 1 μ (=0·001 mm.) of mercury the mean free path of oxygen, the gas in use, is at 0° C. of the order 10 cm. This is large compared with the diameter of the vacuum vess l, and the size of the suspended bodies, so that the density of the mass coming up to the suspended body is unaffected by the motions. For almost every particle reaching the body comes from the bounding surface of the vacuum vessel, and its velocity will depend on the temperature of the bounding surface, so that the distribution of paths will be always equally irregular.

Divide the mass of gas particles in front o the suspended body into six streams. The stream moving towards the body, supposed a flat plane, has velocity V, say $\rightarrow$; the stream moving away from it has velocity $V+2v$ $\leftarrow$, where v is velocity of the plane, and the mass moving to the plane per unit area per second $=\rho(V+v)/6$, ρ being the gas density.

The momentum imparted per second, *i.e.*, the pressure, is :—

$$\rho(V+v)2(V+v)/6.$$

This is practically

$$\rho(V^2+2Vv)/3.$$

On the back face the pressure is :—

$$\rho(V^2-2Vv)/3.$$

The excess on the front face is :—

$$4\rho Vv/3.$$

If s=area of the plane, and a=its arm, $v=a\dot{\theta}$; and we have as equation of motion :—

$$I\ddot{\theta}+\tfrac{4}{3}\rho Vsa\dot{\theta}+\mu\theta=0.$$

Putting this as :—

$$\ddot{\theta}+K\dot{\theta}+n^2\theta=0,$$

where $$K=4\rho Vsa/3I.$$

Solving the above differential equation we obtain :—

$$\text{log dec., } \lambda=KT/4,$$

where T=periodic time.

In September, 1910, when discussing my gravitation research with the late Prof. J. H. Poynting, I said that I thought of measuring the vacuum by observations of damping. Next day Prof. Poynting, with his well-known kindness, sent me the above proof, unsolicited, and his concluding remarks were : " I think the formula $K=4\rho Vsa/3I$ is only likely to be wrong by some factor like π, or $\sqrt{3}$, or $\sqrt{2}$, or some combination thereof. If $KT/4$ is of anything like the order 1/10,000, I doubt whether the air damping can be disentangled from the fatigue of the suspending fibre." In a subsequent letter Prof. Poynting wrote that he had just found that something like a similar solution had been already obtained by Sir G. G. Stokes. Prof. Poynting made it clear that his proof was only approximate, and that he did not see how to work the problem accurately. In thanking Prof. Poynting, I said that if I published this solution I should, of course, acknowledge the authorship, so I take it that he would not have objected to my mentioning his name as I have done, in connection with this proof.

Before proceeding to apply the formula, it should be remarked that the numerical value of $KT/4$ in the following cases is much greater than 1/10,000, and so the criterion above is satisfied.

It is seen that from the above expressions

$$\rho=3I\lambda/VsaT,$$

$$\therefore \text{ the vacuum pressure, } P \text{ (in mm.)}=\frac{760}{1{\cdot}28\times10^{-3}}\cdot\rho.$$

In these experiments V (*i.e.*, velocity of mean square)

$$=5\times10^4, \text{ for oxygen at } 17^\circ\text{C.}$$

$$\therefore P=35{\cdot}6\cdot\frac{I}{saT}\cdot\lambda.$$

In any given experiment I/saT is constant. But with lapse of time, as the vacuum deteriorates after the sealing off, λ will increase, so, for convenience, we have :—

$$P=(35{\cdot}6)\,.\,C\,.\,\lambda,$$

C being a constant.

To quote an example : Readings were taken over a period of five weeks in 1914, the data are :—

$a=1{\cdot}6$ cm.
$I=7{\cdot}2\times(1{\cdot}6)^2$ gm. cm.^{2}.
$s=145\times0{\cdot}07$ cm. (Silver chains were used for the masses carried by the torsion beam, the wire diameter being 0·7 mm.)
$T=132$ sec.
log. $C=\bar{2}{\cdot}0343$.

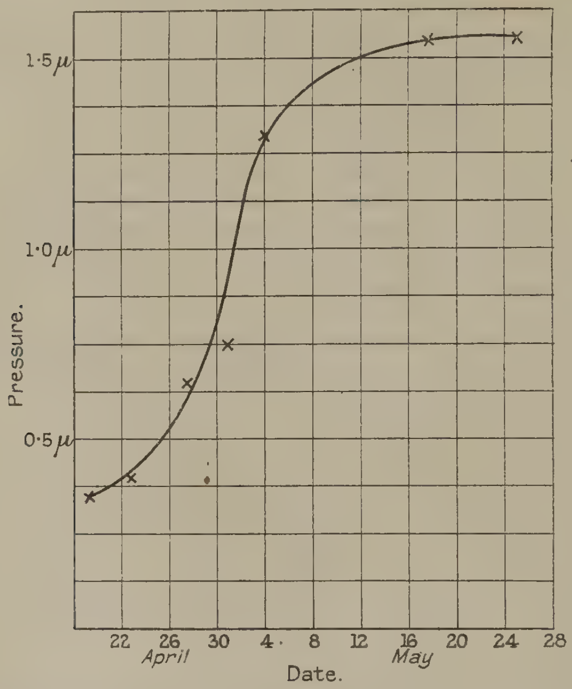
From these figures and the readings of amplitude we derive, from the last equation, values of P, as in the table below and curve on following page :—

Date.	λ.	*P*.
19, IV., 14	0·0009	0·35 μ
23, IV.	0·0011	0·40 μ
28, IV.	0·0016	0·65 μ
1, V.	0·0023	0·75 μ
4, V.	0·0034	1·30 μ
18, V.	0·004	1·55 μ
25, V.	0·004	1·55 μ

At the end of the five weeks considerable damping occurred in the torsion beam oscillations, so that the tube was opened and re-exhausted, and a new series of readings taken. The vacuum for which these results are given had been prepared during four days of heating and pumping, a total of 55 hours' exhausting, during which the whole system, except the Gaede pump, had been kept at over 100 deg. C. A large carbon tube, included in the vacuum, had been at 350 deg. C. for the whole time. At the end of evacuation the tube was well washed out with oxygen. It was then sealed off and allowed to cool. The carbon, let down to room temperature and kept connected to the vacuum, would then absorb residual gases.

1 μ of mercury is used throughout as the unit of pressure for high vacua, as being more convenient than a lengthy decimal fraction of 1 mm. of mercury.

The relation of gas pressure to damping in a vacuum has been investigated by J. L. Hogg (Amer. Acad. "Proc." XLV., pp. 3-7, 1909), who obtained an empirical expression from observations of both pressure and damping.



This method could not readily be applied in my experiments, for it would involve extensive subsidiary experiments.

ABSTRACT.

In experiments on the Newtonian Constant ("Phil. Trans.," May, 1916) the author used a torsion balance in a vacuum which varied in different cases from 15 mm. to 0·00001 mm. pressure. Before sealing the vessel the pressure was determined by a McLeod gauge. Values of the pressure after sealing off were deduced, in the case of the higher vacua, from observations of the damping of the torsion system.

The formula employed is due to the late Prof. Poynting, and can be expressed in the form—

$$P=35{\cdot}6\,\frac{I}{saT}\lambda,$$

where I=moment of inertia of suspended system, s=area of surface (supposed plane) which is experiencing the resistance, a=mean

distance of plane from centre of rotation, T=period of oscillation, and λ=the observed logarithmic decrement.

A table and curve are given showing the relation between P and λ.

DISCUSSION.

Mr. F. J. W. WHIPPLE (communicated remarks): Dr. Shaw's quotation from Prof. Poynting's letter is in the nature of an invitation to a search for missing factors in the formula for the resistance to the motion of a solid in a rarefied gas. It appears to me that there are two reasons for the introduction of such factors. In the first place, the mean velocity of the particles of the gas is not the same as the root-mean-square velocity. If Prof. Poynting's argument is modified to allow for this distinction, the velocities being supposed to be distributed according to Maxwell's law, the formula in question becomes, I believe,

$$K=4\sqrt{\frac{2}{3\pi}}Vsa\rho/I.$$

Some further modification in the formula would seem to be necessary if it is to be applied to the motion of a cylinder. The particles will rebound from a cylinder and a plane lamina in quite different ways, and it is not to be supposed that the resultant force on a cylinder is equal to that which would be experienced by a lamina occupying the central section. May I be allowed to add a word in favour of the use of absolute units for the measurement of pressure. In the theoretical part of this Paper the pressure is given in absolute measure—*i.e.*, if the C.G.S. system is used, in dynes per square centimetre ; but when the results of experiment are under discussion there is a transition to micrometers of mercury. For meterological work the millibar, 1,000 dynes per square centimetre, is now in general use. As it happens that the pressure due to a millimetre of mercury under the standard conditions—*i.e.*, at the fusing point of water at sea level in latitude 45° is very nearly four-thirds of a millibar—the change of units is easily effected. The pressures shown in Dr. Shaw's table run from 0·47 to 2·07 dynes per square centimetre or microbars.

Dr. SHAW communicated the following reply : The value of K, according to Mr. Whipple, should be raised by multiplying it by about $\sqrt{2}$. This possibility was foreseen by Prof. Poynting, and is mentioned in the text of the Paper. The forces due to molecular rebound on (*a*) a plane lamina area A, and on (*b*) a cylinder whose aerial section has area A, are not identical, but I imagine that (*a*) and (*b*) would give results differing by less than, say, 20 per cent., so that for this approximate solution we need not discriminate between the two cases. Any plea for uniformity in our symbols and for the C.G.S. system is well-nigh irresistible, especially where, as here, the results are not accurate enough to preclude the use of the simple multiplier 4/3. Still, an experimentalist measures pressures in millimetres of mercury both on barometer and McLeod gauge. For exact conversion to C.G.S. units one remembers that 1 atmos. pressure is not accurately 10^6 dynes/cm.^{2}, and that 1000/760 is not accurately 4/3 ; and there is some excuse for the experimentalist if he forbears to do the troublesome conversion from the measuring units to the ideal units, and leaves that process to the philosopher on those rare occasions when the conversion is required.

XII. *A Diffraction Colour Box.* By ARTHUR W. CLAYDEN, *M.A., Principal of the Royal Albert Memorial University College, Exeter.*

RECEIVED JANUARY 11, 1917.

THE apparatus described in the following pages was constructed with the object of ascertaining whether the known phenomena of colour vision were based upon any numerical relations between the oscillation periods of the waves of light concerned; and, if so, of endeavouring to determine those numerical relations.

For the purpose in view it was necessary to have some form of apparatus which would yield pure rays whose vibration periods or whose wave-lengths could be easily read with sufficient accuracy, and which would admit of two or three rays of known periods being easily superposed and combined.

Neither Maxwell's colour box nor Abney's colour patch apparatus seemed quite suitable, and after numerous preliminary experiments with various arrangements a device was adopted which meets all requirements, and has the advantage of very great simplicity of manipulation.

It is a combination of the principle of the concave grating with Maxwell's colour box.

A silver on glass speculum by Calver, 6 in. in diameter, and 4 ft. focus, was removed from a telescope, and supported in a vertical position on adjusting screws, as at M in the diagram.

An adjustable slit was supported at S at the centre of curvature of the mirror, so that the image of the slit should be formed on the slit itself.

A Thorpe grating of 14,475 lines to the inch on parallel glass was placed in front of the mirror, so that the grating pressed gently against the silver surface of the mirror, and the rest of the mirror was covered with a diaphragm of black card.

This arrangement is found to give well defined spectra on both sides of the central image in sharp focus along the circumference of a circle, of which the slit and mirror form the ends of a diameter.

The position of the grating was carefully adjusted, so that the spectra were spread out in a horizontal plane.

The spectrum of the first order on one side was selected for use.

The accompanying diagram explains the construction of the apparatus.

S is the slit and G the grating, each carried on a stand which permits the necessary adjustments for exact focus and parallelism between the slit and the lines of the grating.

After getting accurate focus the distance G S was divided equally at P.

A wooden arc at A A′ shows the position of the spectrum.

Two equal arms P B and P B′ are pivoted at P, and carry sliding carriages C and C′, which move smoothly on the arc A A′.

Brass pivots are fixed to each sliding carriage, so that the axis of the pivot coincides exactly with the arc on which the spectrum is in focus.

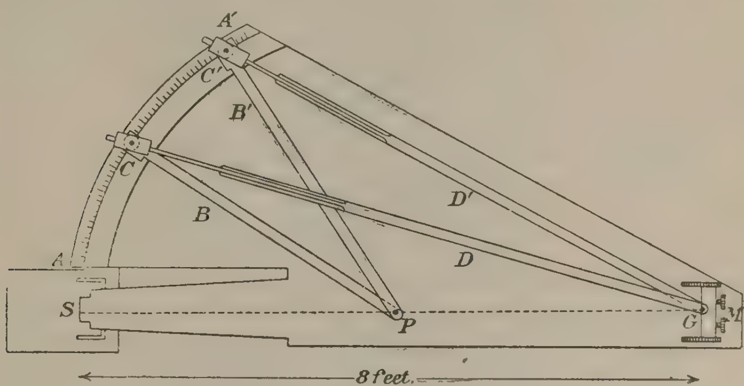


DIAGRAM OF THE APPARATUS.

A telescope eye-piece attached to the sliding carriage and focussed for one part of the spectrum would be in focus for all parts, but in order to see the spectrum properly it would need to be directed towards the grating. This is automatically effected, thus :—

Two long light rods D D′, are pivoted on an axis immediately below the grating, and are attached at their other ends by tubes sliding smoothly to upper portions of the carriages C C′, so constructed as to swing steadily on the brass pivots.

The eye pieces are attached to these upper portions.

The action of the apparatus is as follows : The arms P B, P B′, constrain the sliding carriers to slide on the correct arc, and the rods D D′ oblige the eye-piece to turn always to the

grating, so that if any part of the spectrum is brought to focus in either eye-piece, any other part of the spectrum may be observed by simply sliding the carriage from point to point.

The whole apparatus is encased in a light proof covering (except where the eye-pieces move) made of a wooden framework covered with thick brown paper lined with black cotton cloth. Pieces of the same black cloth are suspended inside the framework to act as numerous diaphragms, and the moving parts are separated from the path of the rays by a division of black cloth.

Sliding black curtains are provided on each side of, and between, the sliding carriages, so that practically no extraneous light can disturb the results.

The use of the Thorpe replica on parallel glass is probably responsible for the formation of a second spectrum, which is not in good focus, but in my apparatus there is an upper spectrum of excellent definition showing a very large number of the Fraunhofer lines from the A group up to the extreme violet.

The other, fainter, spectrum falls below this, and the eye-pieces are so placed as not to see it at all.

Graduation of the Apparatus.

The first step was to graduate the arc C C′.

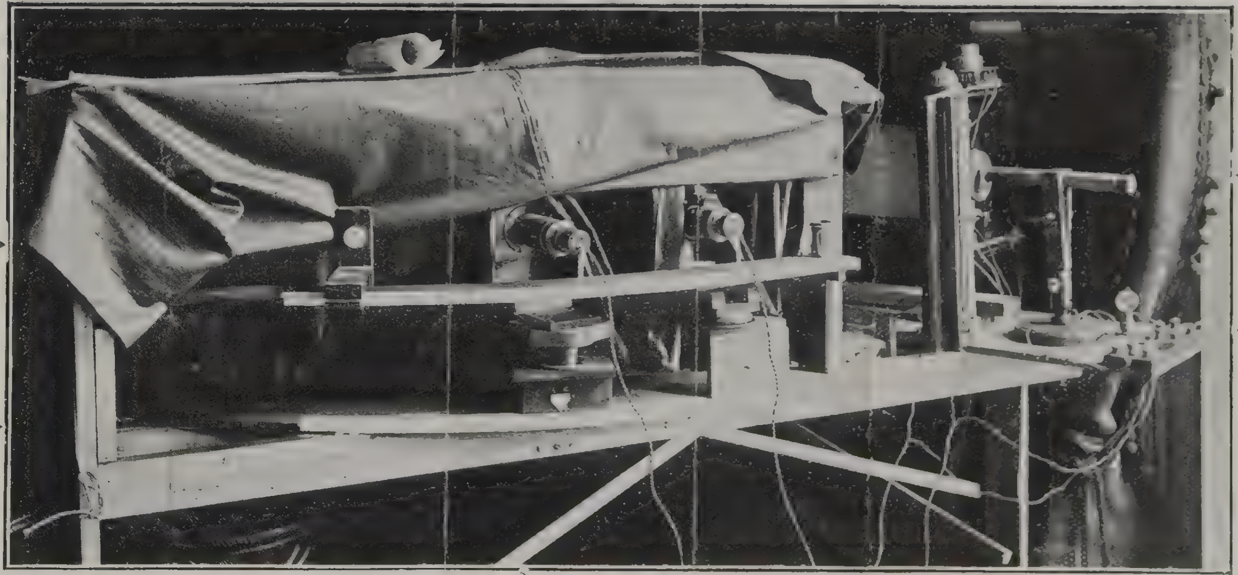
A paper strip was attached to the arc and a mark affixed to the slider C in line with the axis on which its upper part moved.

A considerable series of spectral lines was then observed, taking lines of known wave-lengths from the flame spectra of sodium, lithium, strontium, indium and potassium, the vacuum tube spectra of hydrogen and mercury, and the spark spectra of magnesium and zinc.

The position of the mark on the carrier when each line was central in the field of view was marked on the paper, and found to be proportional to the wave-length.

The dispersion of the apparatus may be gathered from the facts that the sodium D lines are about $\frac{1}{10}$ in. apart; there is approximately 1 in. between the red lines of lithium and hydrogen, and the whole spectrum is about 22 in. long.

Having made a series of observations with one carrier the second was brought into use and centred on the hydrogen red line. A mark was then made on the carrier coincident with the mark on the scale, and on moving the carrier it was found to read the same as the first in all parts of the spectrum.



The Apparatus as it stands in my study. When in use the curtain is dropped over the whole front to cover the sliding carriers, eye-pieces and lamps ; and the window on the right of the picture is closed with shutters.

[*To face page* 178.

As it seemed desirable to deal rather with vibration frequencies than with wave-lengths the next step was to prepare a second scale giving such values.

The oscillation frequencies employed in recent spectroscopic work are inconveniently large numbers, and give the number of waves in a given space. It seemed preferable to deal with the number of oscillations in a given time.

The number of oscillations per second was then computed for a number of different wave-lengths taken at intervals along the spectrum, and the results were plotted in the form of a curve from which the periodicity of any wave-length could be read.

The number of oscillations per second varies from about 39×10^{13} in the extreme red up to about 76×10^{13} at the other end of the visible spectrum.

Observation soon showed that the smallest change of wave-length that could be recognised by the eye as a change of colour was greater than that which corresponded to a change of period of 10^{12} vibrations per second, or to a change of one vibration more or less in $1/10^{12}$ second.

If we take this as our unit of time we get the vibration frequencies of visible light expressed by a series of three figure numbers easily comparable with those expressing the vibrations of sound.

The unit $1/10^{12}$ second might be called a twelfth-second after the analogy of the tenth-metre.

Having the curve showing wave-lengths in terms of vibration periodicity per twelfth-second another scale was constructed and fitted to the wave-length scale on the graduated arc of the apparatus.

The accuracy of the graduation was tested repeatedly and minor readjustments made until it was found to read correctly to within the experimental limits of one vibration per twelfth-second.

Mixtures of Colours.

To study the phenomena presented by the mixture of two or more colours the following method is adopted :—

If a hydrogen vacuum tube is placed before the slit and an induction discharge passed through it, one eye-piece can be so placed as to see the red line and the second one of the other lines, say, the blue.

If, now, an incandescent filament giving white light is placed in the same position in the eye-piece as the image of the red

line, red light of exactly that same period will be returned to the slit. If, then, the slit is opened widely and a small observing telescope is focussed on the grating it will be seen to be brightly illuminated by light of that wave-length only.

Similarly with the blue line, or any other line; and, if both white filaments are incandescent at the same time, the telescope will show the grating illuminated by light of those two periodicities, and none other than those two.

For this purpose some small metallic filament lamps such as are used for motor work are mounted so that they can be slipped into the eye-pieces in such a position that the filament is in the position of the image of a spectral line. The filament is a small Osram spiral whose width is less than half the distance between the sodium D lines.

In the earlier experiments the current for the lamps was drawn from batteries, but subsequently these were replaced by a low voltage current obtained by transforming the domestic supply down to 8 and 5 volts.

The two lamps are connected with the two branches of a divided circuit, one end of which is a sliding contact moving upon a coil of fine iron wire wound on a long bone knitting needle. By altering the position of the contact the relative resistances of the two arms of the circuit can be varied and the intrinsic brilliancy of the lamps easily adjusted at pleasure.

Each lamp circuit also includes a spring tapping key.

A board is fixed under the slit of the apparatus, and this carries the observing telescope, the keys and the sliding contact.

Meanwhile the nearer of the two sliding carriers is easily reached and moved by the left hand.

Hence the lamps can be turned on and off, their relative brilliancy changed, and the colour of the light received from one of them modified, without removing the eye from the observing telescope.

The purity of the light returned to the observing telescope was repeatedly tested by substituting an ordinary spectroscope. The analysis of the light always gave clear, sharply defined lines without any trace of any light other than that of the periodicities indicated by the position of the carriers on the scale.

As it is necessary for some purposes to have a third lamp, one is mounted so that it can be placed by hand in any desired position on the arc, but its support is not provided with the automatic focussing and directing rods.

ABSTRACT.

The apparatus consists essentially of a very simple concave grating spectroscope, of which the slit and grating are situated at opposite diameters of a circle, the spectrum being formed on the arc of this circle. Two independent arms carry fittings on which may be placed either telescope eyepieces or small electric lamps. With the slit of the instrument illuminated by a suitable source, the eyepieces can be set so that any two desired wave-lengths are in the centres of their respective fields of view. The eyepieces are then replaced by the small lamps (the filaments coinciding with the previous positions of the crosslines), and the grating is observed with a small telescope pointed towards the widened slit: the whole of its surface is seen to be illuminated with a mixture of the two colours on which the eyepieces were originally set.

The "concave grating" employed consists of a Thorpe replica of a Rowland plane grating of 14,475 lines to the inch, mounted with its ruled surface in contact with the surface of a concave mirror of 4 ft. focal length. This forms an admirable substitute for the more expensive concave grating.

The author prefers to state results in terms of the number of oscillations per unit of time.

Observations showed that the smallest change of wave-length which could be recognised by the eye as a change of colour was greater than that which corresponded to a change of period of 10^{12} vibrations per second, or to a change of one vibration more or less in $1/10^{12}$ second.

DISCUSSION.

Mr. TROTTER asked how the minimum difference of wave-length detectable as colour difference compared with the values obtained by Dr. Edridge-Green.

Mr. T. SMITH said that experiments on the minimum colour difference which the eye could detect were valueless unless the exact conditions under which the experiments were made were stated. For example, the comparison of two colours side by side is materially affected by the presence of a black space between them. In Dr. Edridge-Green's experiments the conditions were not good. The experiments were made somewhat as follows:—While viewing the extreme red end of the spectrum, a shutter is brought down towards the red so as to cut off everything above the region to which the observer would give one distinguishing name, such as deep red. A second shutter was then brought up from the red end to cut off the first patch, and the first shutter moved along the spectrum until the boundary of the next "monochromatic" patch was located. By this means the whole spectrum was divided up into about 25 monochromatic patches. It is not difficult to repeat these observations, but if they are compared with measurements made in another way entirely different results are obtained. Lord Rayleigh was able, for instance, to distinguish between the colour of the two sodium lines. He had two spectra, one above the other, the upper one being inverted, so that D_1 and D_2 of one spectrum coincided with D_2 and D_1 of the other. Colour comparisons should always be made from simultaneous observations of the two colours.

Prof. C. V. BOYS said he would like to emphasise Mr. Smith's remarks about the effect of varying conditions on the visual estimates of colour. In the colours of soap films, for instance, this difficulty is met with.

Where the thickness is nearly zero the colour is black. As the thickness increases the colours of the first order are successively encountered. Between the blue and yellow there is a tint which can only be described as a bad white. He had long been puzzled as to what this colour was, but had eventually discovered a method of overcoming the deceptive influence of the adjacent colours. Usually these films are traversed by long thin folds. The thickness of the folded film is everywhere three times that of the adjacent unfolded film. Points can be found where this doubtful region between the blue and yellow, occurring on the triple film, can be directly contrasted with the first order white in the adjacent single film. It is then seen to be a very good green. A point which had surprised him in connection with Mr. Clayden's apparatus was that the plane grating, which could not be in actual contact with the mirror except at its edges, nevertheless behaved as if it were in good optical contact.

Dr. W. Eccles pointed out that the minimum colour difference estimated by Mr. Clayden to be perceptible to the eye, if expressed in the same way as an interval in music, works out as about 1·0013. The tone difference just perceptible to the trained musical ear has been stated to be one seven-hundredth of an octave, which works out as 1·001. It is interesting to observe that the sensitiveness of the eye to pitch differences is, like its sensitiveness to inflow of energy, of the same order as that of the ear.

Mr. Clayden, in reply, said that in making his experiments on the minimum difference of wave-length required to produce a noticeable change of colour he had arranged matters so that by using a reversing or non-reversing eyepiece in the observing telescope one or other colour was uppermost in the field of view. The difference in wave-length was then reduced until it was impossible to say which was which. By this method he could detect a difference of two vibrations per $1/10^{12}$ sec., but at the red and violet ends of the spectrum the detectable difference greatly increased. With regard to the President's last remarks, he had himself been very agreeably surprised to find that the performance of the grating was so satisfactory.

An Arrangement for Showing, by Projection on a Screen, the Coloured Fringes Produced when a Transparent Grating is Laid with its Ruled Face against a Plane Mirror, was shown at the Meeting on March 9th, by Mr. A. W. CLAYDEN.

THE light from a small lantern is incident in an approximately parallel beam on the interface of the grating and mirror, which are held together in a wooden framework supported on a stand. The position of the grating-mirror combination is adjusted so that the central reflected beam, or any one of the diffracted beams, is returned just alongside the lantern. The beam passes through a projection lens which focuses the fringes on a distant screen. The fringes thus obtained are very bright, and the simplest apparatus and adjustments suffice for their production.

DISCUSSION.

Dr. R. S. WILLOWS mentioned that Fellows had informed him that these fringes had been described by Barus in the "Philosophical Magazine" (July, 1910, and July, 1911). He understood that they had also been exhibited by Rheinberg at the Optical Convention in 1910. Mr. Clayden's method of exhibiting them by projection would, he thought, be distinctly useful to teachers.

An Apparatus for Studying the Effect of Hertzian Waves on the Beating of the Heart was Exhibited at the Meeting on March 9th by Prof. W. M. Coleman.

A simple pendulum, consisting of a cylindrical brass bob terminating in a pointed wire coaxial with the bob, hangs by a piece of string above one of the terminals of an induction coil, so that in its lowest position the point of the bob is within sparking distance of the terminal and vertically above it. The bob is connected by a piece of flexible wire to the other terminal of the coil. When the pendulum is set oscillating there is a shower of sparks every time the bob passes its lowest position. The frequency of intermittence can be varied by altering the length of the suspension. By adjusting the period of the pendulum nearly to the time of a heart-beat any possible effect on the rate of the beating may be observed. Prof. Coleman stated it to be his experience that the heart beats tend to acquire the same rate as the sparking.

The condensed discharge from two Leyden jars is employed.

XIII. *A Note on the Derivation of the General Equation for Wave Motion in an Elastic Medium.* *By* J. A. FLEMING. *M.A., D.Sc., F.R.S.*

RECEIVED MARCH 1, 1917.

IN order that a wave may be propagated in a medium the latter must possess the power of storing energy in each element of volume in two ways—viz., as potential energy or energy of strain in its most general sense, due to some type of displacement made against an elastic resistance; and also as kinetic energy in virtue of the possession of some quality comparable with density, such that change of displacement bestows on the element energy, whilst the strain is changing. The elasticity of the medium is measured by the ratio of stress to strain involved in the displacement. A wave consists in a periodic transformation of the energy stored per element of volume of the medium from kinetic to potential and vice versa, in such fashion that at points lying on a line called the direction of wave propagation the energy continuously varies in form from point to point; being wholly kinetic or wholly potential at the same moment at certain equidistant points at intervals called half a wave length; whilst at intermediate points it is partly in one form and partly in the other. The characteristic of wave motion is therefore that a certain physical state in the medium, which may generally be called a strain, varies from point to point in a cyclical manner along certain lines at the same instant, and also varies at the same point in a cyclical manner from instant to instant.

If therefore, we choose two closely adjacent points in the line of propagation it is possible to say that at the same instant the difference in the states of the strain at these points is precisely the same as the difference between the states at one of these points at some two successive closely adjacent intervals of time.

This strain may be regarded as produced by a corresponding type of stress, and since the quotient of stress by strain is a measure of the elasticity concerned, we can say that the product of strain and elasticity is a measure of the stress producing that strain. The stress may also be measured in another manner as the product of the mass per unit volume—or density of the medium—or equivalent constant and the strain accelera-

tion or rate of change of the displacement or strain. If we then express in mathematical language this fact, we arrive at a differential equation the solution of which gives us all possible information about the wave motion.

Let us take first the simple case of wave propagation in an elastic material contained in a rigid tube, no friction existing between the material contained in the tube and its walls. Let the tube be of uniform cross-section of unit area. Let m be the density of the medium or kinetic energy factor, and let e be the elasticity or potential energy factor. Let φ denote some form of strain in the material which is propagated through it, whether longitudinal or lateral displacement, torsion, shear or compression. Let $\dot{\varphi}=d\varphi/dt$ denote the *strain-velocity* or time rate of change of displacement, and $\ddot{\varphi}=d^2\varphi/dt^2$ denote the *strain acceleration* or rate of change of displacement at the same point. Let r be the distance along the tube axis measured from some origin, and let two parallel right sections be taken at distances r and $r+\delta r$.

Let φ be the strain in the cross section at distance r and $\varphi+\delta\varphi$ at $r+\delta r$.

Then the proper measure of the wave-making strain at the distance r is the limiting value of the ratio $\delta\varphi/\delta r$—viz., $d\varphi/dr$. This is clear from the analogy with a bar of length l, increased in length by δl by any applied stress, where $\delta L/l$ is called the longitudinal strain, and this last multiplied by Young's modulus is the stress causing that strain.

Hence, if e is the elastic coefficient, the *wave-making stress* or effective stress at the same point will be $ed\varphi/dr$, and the quantity

$$\frac{d}{dr}\left(e\frac{d\varphi}{dr}\right)\delta r$$

will represent the difference between the wave-making stress at r and that at $r+dr$. This change in the stress may be also expressed by the product of the mass of the intermediate section and the strain acceleration—viz., by $m\delta r\ddot{\varphi}$.

Accordingly, if e is independent of r, we have the equation—

$$m\frac{d^2\varphi}{dt^2}=e\frac{d^2\varphi}{dr^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$\frac{d^2\varphi}{dt^2}=\frac{e}{m}\frac{d^2\varphi}{dr^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1\text{A})$$

Which is the well-known differential equation for such plane waves.

The solution of the equation (1) is :—

$$\varphi=f_1\left(r-\sqrt{\frac{e}{m}}t\right)+f_2\left(r+\sqrt{\frac{e}{m}}t\right), \quad . \quad . \quad . \quad (2)$$

as can easily be found by trial.

This equation indicates a wave moving with velocity $\sqrt{e}/\sqrt{m}$, having a wave form whose profile has the equation $\varphi=f(r)$, or a certain function of r.

For $\varphi=f(r)$ denotes a certain curve, of which the abscissa is r and ordinate φ, the curve being at rest. But $\varphi=f\left(r-\sqrt{\frac{e}{m}}t\right)$ denotes the same curve moving bodily forward without change of form with a velocity $\sqrt{e}/\sqrt{m}$, since φ remains the same in the last expression if for r we put $r+r'$, and for t we put $t+t'$, provided that $r'/t'=\sqrt{e}/\sqrt{m}$.

In other words the ordinate of magnitude φ arrives at a point at a distance r' further on at a time later by t'.

Accordingly a disturbance of any kind made in an elastic-dense non-viscous medium filling a rigid tube is propagated with a velocity $\sqrt{e}/\sqrt{m}$, and without change of wave-form, if we assume the absence of friction between the elastic medium and the walls of the tube.

We can next derive in the same simple manner the differential equation for wave motion in a homogeneous, infinitely extended, elastic, dense medium due to a disturbance made at some point taken as origin, if we assume that the elasticity e of the medium and its density m are constant and independent of the distance from the origin.

Describe round the origin two concentric spheres of radii r and $r+\delta r$ respectively. As before, let φ be the value of the strain at distance r and $\varphi+\delta\varphi$, that at $r+\delta r$ at the same instant. Then the wave-making strain at all points of the sphere of radius r is $d\varphi/dr$, and the wave-making stress is $ed\varphi/dr$. This last expression is, however, the stress per unit area, and hence the total stress for the whole spherical surface of radius r is $4\pi r^2 e\frac{d\varphi}{dr}=4\pi e r^2\frac{d\varphi}{dr}$. The increment in this in passing from radius r to radius $r+\delta r$ is $4\pi e\frac{d}{dr}\left(r^2\frac{d\varphi}{dr}\right)\delta r$.

The mass of the spherical shell of thickness δr is $4\pi r^2 m\delta r$, where m is the density and hence, equating the two expressions for the total stress, we have :—

$$\frac{d^2\varphi}{dt^2}m4\pi r^2\delta r=4\pi e\frac{d}{dr}\left(r^2\frac{d\varphi}{dr}\right)\delta r, \quad . \quad . \quad . \quad . \quad (3)$$

or

$$m\frac{d^2\varphi}{dt^2}=\frac{e}{r^2}\frac{d}{dr}\left(r^2\frac{d\varphi}{dr}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or, if e is independent of r :—

$$m\frac{d^2\varphi}{dt^2}=e\left(\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If the rectangular co-ordinates of the extremity of the radius r are x, y and z, so that,

$$r=x^2+y^2+z^2,$$

then it is easy to show that if φ is independent of angular direction,

$$\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}=\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and, therefore, we have from (5) and (6)—

$$\frac{d^2\varphi}{dt^2}=\frac{e}{m}\left(\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz}\right). \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$=\frac{e}{m}\Delta^2\varphi$$

and this is the well-known wave equation for space waves.

The solution of (5) and, therefore, of (7) is easily obtained as follows :—

We note that

$$\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}=\frac{1}{r}\frac{d^2(r\varphi)}{dr^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

as can be found by differentiation. Hence, since r is independent of t, we can write (5) in the form

$$\frac{d^2(r\varphi)}{dt^2}=\frac{e}{m}\frac{d^2(r\varphi)}{dr^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

But the solution of this is

$$r\varphi=f_1(r-ct)+f_2(r+ct), \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where $c=\sqrt{e}/\sqrt{m}$.

Accordingly the solution of (5) and (7) is

$$\varphi=\frac{1}{r}F_1(r-ct)+\frac{1}{r}F_2(r+ct), \quad . \quad . \quad . \quad . \quad (11)$$

where f and F stand for some function of the quantity r ct.

This shows that the waves travelling outward from the source attenuate in amplitude inversely as the distance, but travel without change in wave-form, with velocity $c=\sqrt{e}/\sqrt{m}$. In the above case we are dealing with a single spherical wave originating in a point source.

On showing this method of arriving at equation (7) to Prof. J. W. Nicholson he suggested to the author that the method could be adapted to the case of waves generally, not limited to the above simple point source instance, by the application of Huyghen's princ ple. The transformation given in equation (6) above, viz. :—

$$\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}=\Delta^2\varphi$$

is, of course, only true when φ is a function of r and not of angles or orientations. The Laplacean operator

$$\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}=\Delta^2$$

is, however, an invariant with regard to origin—that is, it retains the same mathematical form if for x, y and z we substitute $x+\alpha$, $y+\beta$, $z+\gamma$, where α, β, γ are constants. Hence it is independent of a shift of origin.

Huyghen's principle states that when any set of waves affect a point in a medium they cause that point to send out its own spherical disturbance, and the waves from each point therefore satisfy equation (7) with that point as origin. But since the operator Δ^2 is independent of origin in form, this means that the equation is satisfied for every origin. The strains also are additive by the principle of superposition of small motions, and hence the equation

$$\ddot{\varphi}=c^2\Delta^2\varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

is satisfied generally for the whole assembly of waves or for the general wave motion at any place and time, no matter how or where originating. Hence, it is true generally.

Finally, we can apply the same method of building up the differential equation to the case of electromagnetic waves in a pure dielectric and arrive at the familiar Maxwell equation for

this type of wave. In this case we consider that the magnetic permeability μ of the dielectric is the kinetic energy factor corresponding to the density of the medium, and that the reciprocal of the dielectric coefficient K is the elasticity coefficient of the medium. For if D is the dielectric displacement or strain and E the electric force or stress at any point in the dielectric then on a rational system of units $D=KE$ or $E=\frac{1}{K}D$. As above, the wave-making strain is dD/dr, if we assume that the waves originate in a point source at distance r. The wave-making stress is, therefore, $\frac{1}{K}\frac{dD}{dr}$ per unit of surface perpendicular to the direction of r.

The strain-acceleration is $\ddot{D}=\frac{d^2D}{dt^2}$. Hence, for a spherical shell of the medium of radius r and thickness δr, we have the equation as before—

$$4\pi r^2 \delta r \mu \frac{d^2D}{dt^2}=\frac{4\pi}{K}\frac{d}{dr}\left(r^2\frac{dD}{dr}\right)\delta r, \quad . \quad . \quad . \quad (13)$$

or

$$\frac{d^2D}{dt^2}=\frac{1}{\mu K}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dD}{dr}\right)$$

$$=\frac{1}{\mu K}\left(\frac{d^2D}{dr^2}+\frac{2}{r}\frac{dD}{dr}\right)$$

or

$$\frac{d^2D}{dt^2}=\frac{1}{\mu K}\Delta^2 D. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

This assumes that D is a function of r only, and that μ and K are constants. It follows that such spherical electromagnetic wave travels outwards with velocity $1/\sqrt{\mu K}$, and that the amplitude varies inversely as the distance from the origin.

The equation (14) is usually obtained directly from the Faraday law of electromagnetic induction, and the Maxwell-Ampère law by the Maxwell's applications to a pure dielectric as follows :—

By Faraday's law we have—

$$-\mu\dot{H}=\text{Curl } E, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where H is magnetic force and E electric force at any point in the dielectric.

By the Maxwell-Ampère law—

$$K\dot{E}=\text{Curl } H, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

but if D is displacement or electric strain then in rational units $D=KE$. Hence, eliminating H, $\dot{H}$, E, $\dot{E}$ from the last three equations we have—

$$\mu K\ddot{D}=-\mathrm{Curl}^2 D. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

But E, D and H are here non-divergent vectors, as we assume no electrified bodies or magnetic poles are in the field.

For such vectors it is easy to show that the operator $-\mathrm{Curl}^2\ (\quad)$ is identical with the Laplacean operator $\Delta^2\ (\quad)$ (*see* "The Wireless Telegraphists' Pocket Book," J. A. Fleming, p. 45, for simple proof of this).

Hence, (17) becomes in this case—

$$\ddot{D}=\frac{1}{\mu K}\Delta^2 D, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

which is the same equation as (14).

The solution is—

$$D=\frac{1}{r}F(r \mp ct), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where $c=1/\sqrt{\mu K}$, and F is some function.

The above method of arriving at the well-known differential equation (7) for wave motion will probably be found by some students of physics more easy to follow than the methods based on the hydrodynamic equations; whilst it is more general than the method given in most books on acoustics, in which the displacement is limited to the form of a wave of condensation and rarefaction in a gaseous medium. It is also equally applicable as shown to the case of electromagnetic waves.

Added April 30th.

The above treatment assumes an isotropic medium for the wave propagation, but it is easy to modify it for the case of anisotropic media. Consider the case of electromagnetic waves in a dielectric with principal dielectric coefficients K_1, K_2, K_3 in the three axial directions and constant permeability μ. Then, if D_1, D_2, D_3 are the electric displacements in the axial directions corresponding to any general displacement D, and if $(\mathrm{Curl}\ ^2D)_x$, $(\mathrm{Curl}\ ^2D)_y$, $(\mathrm{Curl}\ ^2D)_z$ denote the axial components of the vector Curl ^2D or Curl (Curl D), we can write the three equations true for the axial directions as follows:—

$$\mu K_1\ddot{D}_1=-(\mathrm{Curl}\ ^2D)_x,$$
$$\mu K_2\ddot{D}_2=-(\mathrm{Curl}\ ^2D)_y$$
$$\mu K_3\ddot{D}_3=-(\mathrm{Curl}\ ^2D)_z.$$

But the axial components of the vector Curl D are respectively—

$$\left(\frac{dD_3}{dy}-\frac{dD_2}{dz}\right),\quad \left(\frac{dD_1}{dz}-\frac{dD_3}{dx}\right),\ \text{and}\ \left(\frac{dD_2}{dx}-\frac{dD_1}{dy}\right),$$

and the axial components of the vector Curl ${}^2D$ are, therefore,

$$\frac{d}{dy}\left(\frac{dD_2}{dx}-\frac{dD_1}{dy}\right)-\frac{d}{dz}\left(\frac{dD_1}{dz}-\frac{dD_3}{dx}\right),$$

$$\frac{d}{dz}\left(\frac{dD_3}{dy}-\frac{dD_2}{dz}\right)-\frac{d}{dx}\left(\frac{dD_2}{dx}-\frac{dD_1}{dy}\right),$$

$$\frac{d}{dx}\left(\frac{dD_1}{dz}-\frac{dD_3}{dx}\right)-\frac{d}{dy}\left(\frac{dD_3}{dy}-\frac{dD_2}{dz}\right).$$

Equating these last expressions to $-\mu K_1\ddot{D}_1$, $-\mu K_2\ddot{D}_2$, $-\mu K_3\ddot{D}_3$ respectively, we have the equations—

$$\mu K_1\frac{d^2D_1}{dt^2}=\frac{d^2D_1}{dy^2}+\frac{d^2D_1}{dz^2}-\frac{d^2D_2}{dxdy}-\frac{d^2D_3}{dxdz},$$

$$\mu K_2\frac{d^2D_2}{dt^2}=\frac{d^2D_2}{dx^2}+\frac{d^2D_2}{dz^2}-\frac{d^2D_1}{dxdy}-\frac{d^2D_3}{dydz},$$

$$\mu K_3\frac{d^2D_3}{dt^2}=\frac{d^2D_3}{dx^2}+\frac{d^2D_3}{dy^2}-\frac{d^2D_1}{dxdz}-\frac{d^2D_2}{dydz}.$$

But these equations are identical with those given by Maxwell (*see* "Electricity and Magnetism," Vol. II., second edition, § 794), which he shows lead to Fresnel's expressions for wave propagation in a non-isotropic medium, provided we consider the electrostatic potential to be zero.

ABSTRACT.

The Paper explains a simple method of arriving at the general differential equation for wave motion—viz.,

$$\frac{d^2\phi}{dt^2}=c^2\left(\frac{d^2\phi}{dx^2}+\frac{d^2\phi}{dy^2}+\frac{d^2\phi}{dz^2}\right),$$

where c is the velocity of propagation of the wave. A wave motion consists in the propagation of some form of strain through an elastic dense medium or one having comparable qualities. Corresponding to this strain there is an analogous stress in the medium, and we can express this stress mathematically in two ways—viz., kinetically, as the product of the density, and the time rate of change of the strain, or by $m\frac{d^2\phi}{dt^2}$ per unit of volume, and also statically as the space increment of the expression $e\frac{d\phi}{dr}$, where e is a

coefficient of elasticity and r is a vector denoting direction of propagation. Hence, for plane waves we can write the equation

$$m\frac{d^2\varphi}{dt^2}\delta r=\frac{d}{dr}\left(e\frac{d\varphi}{dr}\right)\delta r,$$

or if m and e are independent of distance, we have

$$\frac{d^2\varphi}{dt^2}=\frac{e}{m}\frac{d^2\varphi}{dr^2},$$

which is the known differential equation of wave motion for plane waves. Its solution is $\varphi=F\left(x\mp\sqrt{\frac{e}{m}}t\right)$, where F is some function. In the same manner for spherical waves originating in a point source we can write the stress equation

$$4\pi r^2 m\delta r\frac{d^2\varphi}{dt^2}=\frac{d}{dr}\left(4\pi r^2 e\frac{d\varphi}{dr}\right)\delta r,$$

which at once gives

$$\frac{d^2\varphi}{dt^2}=\frac{e}{m}\left(\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}\right);$$

or if $r^2=x^2+y^2+z^2$, we have

$$\frac{d^2\varphi}{dt^2}=\frac{e}{m}\left(\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}\right).$$

The solution of this last equation is

$$\varphi=\frac{1}{r}F\left(x\mp\sqrt{\frac{e}{m}}\right).$$

where F is some function.

The general equation for electromagnetic wave propagation in a pure dielectric may also be obtained in the same manner. In this case the strain on the medium is the electric displacement D, and, if E is electric force, then on a system of rational units $1/K$, where K is the dielectric constant, is the electric elasticity. Hence we can consistently assume that the magnetic permeability μ corresponds to density, and then the kinetic measure of the electric stress is $\mu\frac{d^2D}{dt^2}$, and this must be equated to the space variation of the static measure of the electric stress.

For the spherical wave from a point source this leads to the equation

$$\frac{d^2D}{dt^2}=\frac{1}{\mu K}\left(\frac{d^2D}{dx^2}+\frac{d^2D}{dy^2}+\frac{d^2D}{dz^2}\right),$$

with the solution

$$D=\frac{1}{r}F\left(x\mp\frac{1}{\sqrt{\mu K}}t\right).$$

The wave is, therefore, propagated outwards without change of wave form; but the amplitude varies inversely as the distance from the source.

Since the Laplacean operator $\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}$ is an invariant, and

is not altered in form by a shift of origin; and since we can by Huyghen's principle consider a wave of any kind as the resultant of a set of spherical waves originating at various point sources, we can apply the above method to the case of wave motion generally.

The method above described may be epitomised by saying that we obtain the differential equation by equating the product of strain-acceleration $\frac{d^2\phi}{dt^2}$ and density to the static measure of the stress expressed as the space variation of the product of the elasticity and the strain slope $\left(e\frac{d\phi}{dr}\right)$, which is the proper measure of the stress at the point considered.

DISCUSSION.

Mr. F. J. W. WHIPPLE thought that the validity of Prof. Fleming's application of Huyghen's principle might be questioned. In the first place, he thought that the principle itself required proof just as much as the property that it was used to establish. Further, the secondary wavelet from each point in a wave front was assumed to be spherical. Was it not the case that the phase of these disturbances was a function of the direction, relative to the main wave front, as well as of the distance from the origin of the wavelet? Another interesting question was how the equations would have to be modified in the case of the energy reflected at the boundary surfaces of the medium.

Prof. C. H. LEES did not quite see how Prof. Fleming's treatment of the problem differed from that given in Love's "Elasticity," for example.

Prof. J. A. FLEMING, in reply, wrote that the criticisms of the two speakers in the discussion rather missed the point of the Paper. The validity of Huyghen's principle is not in question. Most writers on physical optics have resorted to it, and he (Dr. Fleming) had never seen any explanation of the phenomena of diffraction which did not rest upon it. The proof given in the Paper that the characteristic equation proved for spherical waves with origin at the centre may be applied to waves however originating is made to rest upon the fact that the Laplacean operator is invariant with regard to origin. The object of the Paper was not to write a complete theory of wave motion, nor of elasticity, but merely to give a method, which seems valid, for arriving immediately at the characteristic equation in a manner which would appeal to students not very well versed in hydrodynamics or the theory of elasticity. Prof. Lees asks how far the treatment differs from that given, say, in Love's "Theory of Elasticity." The answer is not at all in the result; but the method used in the Paper enables us to arrive by simple considerations at the characteristic differential equation, and then to elucidate the physical meaning of its solution. It seems better for a physical and engineering student to have some approach to such conception rather than none at all. The treatment of the subject of wave motion is generally very difficult to follow in advanced text-books, whereas the practical applications of sound, water and electromagnetic waves makes some theoretical knowledge of the subject necessary.

XIV. *The Effect of Stretching on the Thermal Conductivity of Wires.* By A. JOHNSTONE, *B.Sc.*

RECEIVED MARCH 7, 1917.

COMMUNICATED BY PROF. C. H. LEES, F.R.S.

OUTLINE OF METHOD USED.

1. MOST observers, in determining the thermal conductivity of substances which conduct heat readily, have found it advisable to use the material in the form of wire and to let the heat flow along the axis. This method was adopted in the present experiments. The method is simplified f the heat flows from the centre of the wire to the two ends, which are kept at a constant temperature. In the following experiments the wire was surrounded by a vessel to ensure this, heat was supplied to the centre of the wire by passing a current of electricity through a manganin coil wound on the wire, and the temperature difference between two points on the same side of the centre was ascertained by means of two platinum coils also wound on the wire.

THE APPARATUS.

2. The wire to be tested AA, (Fig. 1) was fixed at each end between the jaws of screw clamps, B, joined to the brass cylinders, through which water could be circulated. To one clamp was attached a rod, T, passing through a framework, so that the tension could be applied by a nut working on a thread, T'; and to the other was attached a spring balance, K, by which the tension was measured.

The heating coil H_t was made from 14 cms. of No. 40 manganin wire closely wound on cylindrical sleeves of micanite, which could just slide on to the wire; 0·5 cm. of the wire was left unwound at each end, and to these ends were soldered covered copper leads of No. 30 gauge. The platinum thermometers (H, C, J') used were made from No. 40 single silk-covered platinum wire each of 35 cms. length, wound like the heating coil and having similar leads. This gave them a resistance of about 3 ohms at ordinary temperatures. The heating coil was placed at the centre of the wire under examination which was about 16 cms. long; one platinum thermometer was placed near to it, and the other near one of the clamps. For purposes of reference the former will be called the "hot coil" and the other the "cold coil." To support the leads from the

coils against breakage, each coil was tied by silk threads to a light wooden strip which it touched at two points and to which the leads were fixed. The coils were kept at a constant dis-

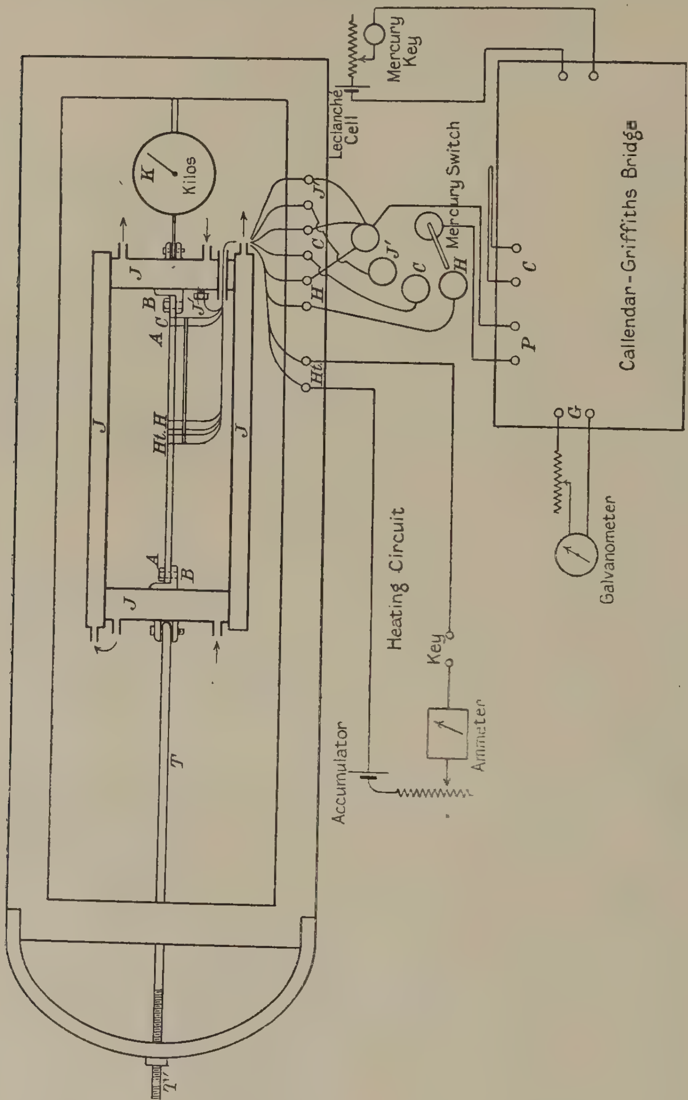


FIG. 1.—APPARATUS AND SCHEME OF CONNECTIONS.

tance apart by a cross strip of mica and were strengthened by a coating of shellac to prevent collapse during subsequent removal. The insulated leads from the coils were passed through

a brass tube in one of the end clamps to mercury cups in the wood frame, and the wire and end pieces were surrounded by a water jacket. Inside and touching the jacket was a supplementary platinum coil to enable changes in the jacket temperature to be detected.

The resistances of the coils were determined by means of a Callendar-Griffiths bridge, which was read in ohms to the fourth decimal place. For the galvanometer used a deflection of 1 mm. on the scale represented a difference of 0·0002 ohm, and for the coils used this would represent a temperature difference of about 0·01°C. For the conditions of the experiments performed this would enable a difference in the conductivity of 0·05 per cent. to be detected. The current to the heating coil was supplied by an accumulator and was regulated by an adjustable resistance. The current for the bridge was supplied by a Leclanché cell in series with an adjustable resistance. Each of the platinum coils could be connected in turn with the bridge at P through the mercury key as shown in the diagram.

Theory of the Method Used.

3. If a thin rod of uniform cross section is entirely surrounded by air in a vessel kept at constant temperature, while heat is supplied to it near one end and the other is kept in good thermal contact with the wall of the vessel, then *

$$v=\frac{Q}{(phqk)^{\frac{1}{2}}} \cdot \sinh\left(\frac{ph}{qk}\right)^{\frac{1}{2}}x \Big/ \cosh\left(\frac{ph}{qk}\right)^{\frac{1}{2}}x_C \quad . \quad . \quad (1)$$

where v is excess of temperature of rod at the cross section x cm. from the end in contact with the vessel, over that of the vessel, p is the perimeter, q is the area of cross section, h the emissivity, k the thermal conductivity of the rod, and Q is the amount of heat per second crossing the section at any point x_C

If the heat is supplied at a uniform rate per centimetre to the surface of the rod between x_C and the free end, the mean temperature of the surface between x_C and the end may be taken as $v_{C'}$, the temperature which would be observed at a point $x_{C'}$, dividing the distance between x_C and the free end in the ratio 1 : 2, if the equation (1) held throughout the heated

* *See* Lees, Bakerian Lecture, "Phil. Trans." Roy. Soc. A 208, pp. 381, 443 (1908).

part of the rod. If s is the area of the surface of the rod beyond x_C and H is the total heat supplied to the rod, then

$$Q=H-h\,.\,sv_{C'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the temperature excesses v_A and v_B of the rod over the air at two sections x_A and x_B are observed then,

$$k=\left\{\frac{H(x_AS_A-x_BS_B)}{v_A-v_B}-hsx_{C'}S_{C'}\right\}\Big/q\cosh ax_C \quad . \quad . \quad (3)$$

where a is written for $(ph/qk)^{\frac{1}{2}}$ and S_A for $\sinh ax_A/ax_A$, &c.

From equation (3) k may be calculated with sufficient accuracy by taking, $\sinh ax/ax=\cosh ax=1$ approximately, and neglecting the term $\left(\frac{hsx_{C'}}{q}\right)$, which, for the experiments described, introduces an error in k of approximately 0·003 per cent.

We thus get

$$k=\frac{H}{q}\cdot\frac{x_A-x_B}{v_A-v_B}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Since for any one experiment H, q, x_A-x_B, were constant we may take k proportional to

$$(v_A-v_B)^{-1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If R_1 is the resistance of the platinum thermometer used to determine v_A, and r_1 is that for v_B when both are at the jacket temperature, and if R_2 and r_2 are the corresponding values when a temperature difference exists; then it may be shown that

$$v_A-v_B=\frac{1}{B}\left(\frac{R_2}{R_1}-\frac{r_2}{r_1}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where B is a constant.

Hence for any experiment we may take

$$k \text{ proportional to } \quad \frac{1}{q}\left(\frac{R_2}{R_1}-\frac{r_2}{r_1}\right)^{-1}(x_A-x_B)\,(\text{current})^2. \quad . \quad . \quad (7)$$

In order to detect inconsistencies in readings during an experiment a method for finding v_A-v_B approximately was also used.

Method of Experimenting.

4. When taking a resistance reading for any of the coils, to avoid thermo-effects the galvanometer was kept in circuit and the deflection of the galvanometer noted when the Leclanché cell was connected to the bridge. At the beginning of an experiment tap water was circulated through the jacket and the resistance of the jacket coil was read at intervals. After about

45 minutes the temperatures usually became steady. The resistances of the "hot" and "cold" coils were then read. Then a suitable current was sent through the heating coil. After about 30 minutes the resistances of the coils had become steady and were again read. Owing to slight variations in the jacket temperature it was seldom possible to repeat these readings and obtain exactly the same values. Consequently an attempt was made to read the resistances of the two coils with as small a time difference as possible. A number of such double readings was taken and the mean temperature difference of the coil calculated The resistance of the jacket coil was read at intervals to detect abnormal changes. The wire was then stretched by a force about 0·7 of the elastic limit and the resistances of the coils read again. The stretching force was then removed and readings again taken. The temperature difference between the hot and cold coils varied between 10°C. and 20°C.

The wires tested were all (except the nickel one, which was No. 16 gauge) of No. 14 gauge. When new wires were used the results for successive experiments were not consistent with each other until about the third experiment, owing probably to initial stresses in the wires; and the results were more consistent during the summer months, when the temperature of the jacket water was not affected by the heating apparatus. The effect of the stretching on the dimensions of the wire is neglected, as its influence on the conductivity is too small.

5. Comments on the Results.

For all the wires used, stretching produced a slight increase in thermal conductivity. Although difficulty was experienced in getting results which were consistent throughout, those obtained when conditions were most satisfactory show an increase of about 0·5 per cent. for a tension about 0·7 of the elastic limit. A few experiments apparently showed a decrease in conductivity o the order 0·1 per cent. w th stretching, but in these cases the readings were always unsteady and unsatisfactory. In all cases when readings were steady an increase was observed. Several attempts were made to determine the change in conductivity for smaller tensions, and the results seem to indicate that the change is not appreciable until the tension approaches the elastic limit. After the tension was withdrawn the conductivity returned approximately to its original value, although in most cases the value was slightly in excess

Observations and Results.

Date. 1914.	Duration of experiments. Hrs.	Tensions. Kilos.	Tensions. Kilos/cm.^{2}	$\left\{\frac{R_2}{R_1}-\frac{r_2}{r_2}\right\}^{-1}$ proportional to k.	Percentage increase in k.	Current used to give H. Amps.	$\left\{\frac{R_2}{R_1}-\frac{r_2}{r_1}\right\}^{-1}$ $\times$(current)^2 $\times(x_A-x_B)$.
Copper Wire, No. 14 *Gauge.* $x_A-x_B=4{\cdot}25$ cm.							
July 8th...	4	0 15 0	463	26·88 26·98 26·90	0·37	0·3	10·28 10·32 10·29
Sept. 8th .	5	0 10 0	309	16·03 16·12 16·09	0·56	0·4	11·03 11·09 11·07
Sept. 9th .	6	0 10 0	309	15·82 15·84 15·82	0·13	0·4	10·88 10·90 10·88
Sept. 10th.	5	0 10 0	309	27·10 27·29 27·12	0·70	0·3	10·37 10·44 10·38
Sept. 11th.	5	0 10 0	309	27·03 27·25 27·10	0·81	0·3	10·34 10·42 10·37
Steel Wire, No. 14 *Gauge.* $x_A-x_B=4{\cdot}45$ cm.							
Oct. 21st .	$4\frac{1}{2}$	0 40 0	1,235	16·66 16·74 16·69	0·48	0·2	2·966 2·980 2·971
Oct. 28th .	$4\frac{1}{2}$	0 40 0	1,235	16·57 16·77 16·66	1·21	0·2	2·949 2·985 2·966
Nov. 4th .	$4\frac{1}{2}$	0 40 0	1,235	16·36 16·46 16·36	0·61	0·2	2·912 2·929 2·912
1915. *Nickel Wire, No.* 16 *Gauge.* $x_A-x_B=4{\cdot}75$ cm.							
June 23rd.	4	0 15 0	723	25·46 25·51 25·21	0·20	0·16	3·095 3·101 3·066
June 24th.	4	0 20 0	964	29·33 29·40 29·35	0·24	0·16	3·567 3·575 3·569
June 30th.	$3\frac{1}{2}$	0 20 0	964	25·51 25·66 25·50	0·59	0·16	3·104 3·120 3·100

Date. 1914.	Duration of experiments. Hrs.	Tensions. Kilos.	Tensions. Kilos/cm.^{2}	$\left\{\frac{R_2}{R_1}-\frac{r_2}{r_1}\right\}^{-1}$ proportional to k.	Percentage increase in k.	Current used to give H. Amps.	$\left\{\frac{R_2}{R_1}-\frac{r_2}{r_1}\right\}^{-1}$ $\times$(current)^2 $\times(x_A-x_B)$
		Aluminium Wire, No. 14 Gauge. $x_A-x_B=4{\cdot}55$ cm.					
Nov. 18th.	4½	0 15 0	463	14·64 14·68 14·68	0·27	0·3	5·995 6·012 6·012
Nov. 25th.	4½	0 15 0	463	18·66 18·75 18·66	0·48	0·26	5·740 5·768 5·740
Dec. 2nd...	3	0 15 0	463	18·94 19·18 18·79	1·27	0·26	5·826 5·899 5·780
		Brass Wire, No. 14 Gauge. $x_A-x_B=4{\cdot}6$ cm.					
Dec. 16th.	4¼	0 15 0	463	15·51 15·55 15·44	0·26	0·2	2·854 2·861 2·841
Jan. 27th, 1915	4	0 15 0	463	15·20 15·25 15·19	0·33	0·2	2·797 2·805 2·795
1915.		*Zinc Wire, No. 4 Gauge.* $x_A-x_B=4{\cdot}65$ cm.					
Mar. 3rd...	4	0 15 0	463	21·82 21·90 21·72	0·37	0·2	4·059 4·072 4·040
May 5th...	4½	0 15 0	463	21·17 21·23 21·21	0·28	0·2	3·938 3·950 3·945
May 12th.	4	0 15 0	463	21·93 22·03 unsatisfactory	0·46	0·2	4·079 4·098 unsatisfactory
May 19th.	3½	0 15 0	463	22·52 22·68 22·62	0·71	0·2	4·189 4·218 4·207
May 26th.	4	0 15 0	463	22·93 22·96 22·94	0·10	0·2	4·265 4·271 4·267

of the original value. The numbers quoted for k in the results are values proportional to the thermal conductivity.

The relative conductivities calculated from the values of the heating currents and of $x_A - x_B$, on the assumption that the conductivity of copper is 0·90 are: copper 0·90, steel 0·25, nickel 0·18, aluminium 0·50, brass 0·24, zinc 0·35, whose relation is of the right order.

The experiments described in this Paper were carried out at the suggestion of Prof. Lees, to whom I am indebted for the general design of the apparatus used and for valuable suggestions during the course of the work.

ABSTRACT.

In the experiments carried out, the wire to be tested was fixed at each end between the jaws of screw clamps joined to brass cylinders, through which water could be circulated. Attached to one clamp was a rod passing through a framework, enabling tension to be applied by a nut working on a thread.

Heat was supplied to the centre of the wire by passing a current of electricity through a manganin coil wound on the wire, and the temperature difference between two points on the same side of the centre was ascertained by two platinum coils, also wound on the wire.

The resistances of the coils were determined by means of a Callendar-Griffiths bridge, enabling a temperature difference of 0·01°C. to be read, and making it possible to detect a difference in conductivity of 0·05 per cent. If R_1 is the resistance of the platinum thermometer nearer the heating coil, and r_1 is that of the other platinum coil when both are at the jacket temperature; and if R_2 and r_2 are corresponding values when a temperature difference exists, then it may be shown that the conductivity

$$K \text{ is proportional to } \left(\frac{R_2}{R_1} - \frac{r_2}{r_1}\right)^{-1}.$$

For all the wires used (copper, steel, nickel, aluminium, brass, zinc), stretching produced a slight increase in thermal conductivity. The most satisfactory experiments showed an increase of about 0·5 per cent. for a tension of about 0·7 of the elastic limit.

After the tension was withdrawn the conductivity returned approximately to its original value.

DISCUSSION.

Mr. F. E. Smith suggested that it would have been of interest if the author had made simultaneous observations of electrical conductivity on the same apparatus. Had any experiments been made on liquids, such as mercury, for example ?

Prof. Boys asked if the coefficient would be reversed if compression were applied instead of extension. He realised there would be some practical difficulties in making the experiments.

Prof. Lees, in reply, said that there would have been electrical measurements included in the Paper but for the author's absence in the Army. He thought compression measurements could be made, though they had not so far been carried out.

XV. *The Effect of Stretching on the Thermal and Electrical Conductivities of Wires.* By CHARLES H. LEES, *D.Sc., F.R.S.*

RECEIVED APRIL 26, 1917.

THE measurements which have been made of the effect of stretching on the electrical conductivities of wires for currents flowing along them by Tomlinson, Gray, Smith, Credner and others * have all shown that the fractional decrease of conductivity produced is approximately proportional to the tension, and for a force of a megadyne per square centimetre is of the order 10^{-6}. This holds in all cases with the possible exception of nickel.†

For loads up to the limit of elasticity of the material of the wire the change of electrical conductivity which can be produced does not exceed a few parts in a thousand.

When we turn from the electrical to the thermal conductivity we find that few measurements of the effect of stretching on it have been made. When Mr. Johnstone commenced his work on the subject the only information available was that given by Smith‡ for iron, steel, copper and brass. He found that the effect of loads up to the elastic limit was to increase the thermal conductivities by amounts of the order 2 to 7 per cent. The effect of the same load on the thermal was from 4 to 13 times that on the electrical conductivity.

In general the effect of any change in the purity or physical nature of a material has been found to be greater on the electrical than on the thermal conductivity. Thus, the differences in the electrical conductivities of the samples of nearly pure metals used by Jäger and Diesselhorst and by myself § are considerably greater than those in the thermal conductivities.

For alloys Schulze || and Eucken and Gehlhoff ¶ have shown that the decrease in electrical conductivity as compared with the conductivities of the constituents is much more marked

* References up to 1900 will be found in Wiedemann's "Elektricität," Bd. 1, and in Winkelmann's "Handbuch der Physik," Bd. 4. Later references are: Williams, "Phil. Mag." XIII., p. 635 (1907); Ercolini, "N. Cimento," XIV., p. 537 (1907); Smith, "Physical Review," XXVIII., p. 107 (1909); Credner, "Zeits. Phys. Chem.," LXXXII., p. 457 (1913).

† Tomlinson's coefficients of 10^{-6} are, for iron, platinum, copper, aluminium, magnesium and platinum-silver about 2; for silver, zinc and platinum-iridium about 4; for tin, 11; for lead, 17; and for nickel, −3; and subsequent work has not modified these numbers seriously. Wiedemann. "Elektricität," 1, p. 501.

‡ Smith *l.c.*

§ Compare the differences shown in Fig. 3 and plate 30 of the Bakerian Lecture, "Phil. Trans.," A. 208, p. 381 (1908).

|| Schulze, "Ver. d. Phys. Ges.," XIII., p. 856 (1911).

¶ Eucken and Gehlhoff, "Ver. d. Phys. Ges.," XIV., p. 169 (1912).

than the decrease in the thermal conductivity, and that in consequence the quotient of the thermal by the electrical conductivity, which is of prime importance in the electronic theories of conductivity, increases as the alloy deviates in composition from either of its pure constituents.*

If we compare the two ranges of values within which the thermal and electrical conductivities of different materials lie, we find that the range for thermal is much less extensive than that for electrical conductivities. Thus, the thermal conductivities of glass and copper or silver are as 1 to 400, while the electrical conductivities are as 1 to 6×10^{19}. Or, confining ourselves to metals for the present purpose, the thermal conductivities of bismuth and copper or silver are as 1 to 50 while their electrical conductivities are as 1 to 70.

General considerations such as these made me hesitate to accept Smith's high results for the effect of stretching on the thermal conductivities of the metals he used, and induced me to get Mr. Johnstone to measure the effect by the method he has described in the previous Paper. His results confirm those of Smith, as may be seen from the following table calculated from the observations now available :—

Fractional Increase of Conductivity Due to Stretching.

Material.	Stress. $\frac{\text{megadyne.}}{\text{sq. cm.}}$	Thermal conductivity. Observed.	Thermal conductivity. per $\frac{\text{megadyne.}}{\text{sq. cm.}}$	Electrical conductivity. Observed.	Electrical conductivity. per $\frac{\text{megadyne.}}{\text{sq. cm.}}$	Observer.
Copper...	1,580	0·0080	$5\ 10^{-6}$	−0·0025	$-1{\cdot}6\ 10^{-6}$	Smith.
	1,150	0·013	$11\ 10^{-6}$	...	...	Smith.
	303	0·0049	$16\ 10^{-6}$	...	...	Johnstone.
	...	...	...	...	$-2{\cdot}3\ 10^{-6}$	Tomlinson.
Zinc	454	0·0038	$8{\cdot}4\ 10^{-6}$	...	...	Johnstone.
					$-4{\cdot}4\ 10^{-6}$	Tomlinson.
Aluminium	454	0·0067	$15\ 10^{-6}$	...	...	Johnstone.
					$-1{\cdot}9\ 10^{-6}$	Tomlinson.
Nickel ...	946	0·0037	$3{\cdot}9\ 10^{-6}$	...	...	Johnstone.
					$3{\cdot}2\ 10^{-6}$	Tomlinson.
Iron	2,300	0·025	$11\ 10^{-6}$	−0·0031	$-1{\cdot}4\ 10^{-6}$	Smith.
	1,150	0·018	$16\ 10^{-6}$	...	...	Smith.
					$-2{\cdot}1\ 10^{-6}$	Tomlinson.
Steel	5,030	0·026	$5{\cdot}2\ 10^{-6}$	−0·0018	$-0{\cdot}4\ 10^{-6}$	Smith.
	2,300	0·030	$13\ 10^{-6}$	...	...	Smith.
	1,210	0·0077	$6{\cdot}4\ 10^{-6}$	...	...	Johnstone.
Brass ...	2,590	0·022	$8{\cdot}5\ 10^{-6}$	−0·0028	$-1{\cdot}1\ 10^{-6}$	Smith.
	1,170	0·016	$14\ 10^{-6}$	...	...	Smith.
	454	0·0030	$6{\cdot}6\ 10^{-6}$	...	...	Johnstone.

* It should be noted, however, that Reitzsch, "Ann. der. Phys.," III., p. 403 (1900), found that the addition of small quantities of phosphorus or arsenic to copper reduced its thermal in a slightly greater ratio than its electrical conductivity.

Although the larger of the tensions used by Smith are outside the elastic limits for the materials, there is as fair an agreement as could reasonably be expected between the coefficients of the changes of conductivity per megadyne per square centimetre of stress as calculated from his observations, those of Johnstone for the thermal conductivity and those of Tomlinson for the electrical conductivity.

If we except the change of the electrical conductivity of the hard nickel used by Tomlinson, which was probably far from pure, the results so far available appear to justify the statement that when the commoner metals and alloys are subjected to a tension not exceeding the elastic limit for the material, the thermal conductivity in the direction of the tension is *increased* at a rate between 4 and 10×10^{-6} of itself per megadyne per square centimetre of section, while the electrical conductivity is *decreased* under the same circumstances at about a tenth of the above rate. The quotient of the thermal by the electrical conductivity increases at a rate between 4 and 11×10^{-6} of itself per megadyne of stretching force per square centimetre.

XVI. *Cohesion.* (*Third Paper.*) *By* HERBERT CHATLEY, *D.Sc.*(*Lond.*)

RECEIVED MARCH 15, 1917.

THE objects of this Paper are two-fold:—

1. To restate and add further evidence in favour of an electrical theory of cohesion.

2. To provide certain tentative forms of empirical formulæ for the expression of the inter-molecular forces.

Definitions.

For the purposes of this inquiry, cohesion is defined as follows:—

Cohesion is the net attraction (*i.e.*, balance of attraction over repulsion, between molecules which are relatively chemically saturated, at distances not greatly exceeding the molecular diameters. Adhesion is the special case of cohesion where the molecules are dissimilar in constitution.

The Electrical Theory of Cohesion.

In B, p. 312, it is suggested that "Presumably the enormous electrostatic forces which exist round free atoms are not wholly neutralised in the formation of molecules, and cohesion is due to what electricians call "stray field."

On the score of magnitude the cohesive attraction fades into chemical affinity (electrostatic linkage) as the proximity increases, and diminishes into Newtonian gravitation as the particles are separated.

The "stray field" of a bi-atomic molecule may be conceived in various ways according to the manner of linkage, *e.g.*, with or without an electron as the medium. In any case the whole field must be polarised, and the stray field will be expressible in vector terms. Since the electrostatic bond between two monovalent ions in contact is about 10^{-4} dynes, the cohesive attraction should be less than this. Furthermore, at a distance of about two molecular diameters the field must become almost uniform, and at ten diameters simple Newtonian conditions must exist.

The Magnitude of Electrostatic and Newtonian Linkages.

The following table indicates roughly the magnitude of the linkages conceivable between different simple bodies.

The diameter of an electron has been taken as $3{\cdot}0 \times 10^{-13}$cm.

The mass of an electron as $0{\cdot}61 \times 10^{-27}$ grams.

The diameter of an atom as

$$\sqrt[3]{\frac{1{\cdot}6w}{s}} \, . \, 10^{-8} \text{ cm.}$$

where w is the atomic weight and s is the density at absolute zero.

No.	Description of pair.	Position.	Distance c. to c. Cms.	Electro-static force or chemical affinity. Dynes.	Newtonian force. Dynes.	Ratio.
1	Two electrons	In contact	$3{\cdot}0 \times 10^{-13}$	$-1{\cdot}3 \times 10^{6}$	$2{\cdot}7 \times 10^{-37}$	$0{\cdot}5 \times 10^{43}$
2	Do. do.	1 cm. apart	1·0	$-1{\cdot}6 \times 10^{-19}$	$2{\cdot}5 \times 10^{-62}$	$0{\cdot}5 \times 10^{43}$
3	One positive atom and one electron	In contact	$0{\cdot}7 \times 10^{-8}$	$2{\cdot}0 \times 10^{-3}$	$1{\cdot}3 \times 10^{-42}$	$1{\cdot}5 \times 10^{39}$
4	One positive atom (nascent hydrogen) and one negative similar atom	In contact	$1{\cdot}5 \times 10^{-8}$	$5{\cdot}1 \times 10^{-4}$	$7{\cdot}6 \times 10^{-40}$	$0{\cdot}7 \times 10^{36}$
5	Do. caesium diatomic ions	In contact	$6{\cdot}1 \times 10^{-8}$	$1{\cdot}3 \times 10^{-4}$	$8{\cdot}0 \times 10^{-37}$	$0{\cdot}2 \times 10^{33}$
6	Do hydrogen ions	Gas at N.T.P.	$7{\cdot}6 \times 10^{-6}$	$2{\cdot}0 \times 10^{-9}$	$3{\cdot}0 \times 10^{-45}$	$0{\cdot}7 \times 10^{36}$
7	Do. do.	1 cm. apart	1·0	$1{\cdot}6 \times 10^{-19}$	$2{\cdot}5 \times 10^{-55}$	$0{\cdot}7 \times 10^{36}$

NOTE RE No. 5.—This has been chosen because w/s is a maximum for caesium, and it has been made diatomic so as to indicate the dimensional effect of a molecular interval as distinct from an atomic one.

The mean free path of a nascent atom or a molecule in a gas at N.T.P. is taken as $7{\cdot}6 \times 10^{-6}$ cm.

The Magnitude of the Cohesive Force.

The net value of the cohesive attraction under certain given conditions of expansion is known from tensile tests. Surface tension and the energy of liquefaction also help toward this end. The mean repulsion in the case of a permanent gas also shows what is required to maintain separation.

For iron, in A, p. 447, a molecular linkage of $5{\cdot}0 \times 10^{-8}$ dynes is suggested. In B, p. 310, a value of $2{\cdot}0 \times 10^{-7}$ dynes is employed. If a tetratomic molecule is considered and the ultimate stress is taken as 2,000,000 grammes per square centimetre the value, neglecting reinforcement by adjacent molecoles, is $2{\cdot}5 \times 10^{-6}$. The heat of liquefaction indicates a mean resistance of $1{\cdot}0 \times 10^{-5}$ dynes per molecular pair, if one half the energy is used in producing potential energy, but, of course, this is very approximate.

If the tensile strength of ice is taken as 10 kilogrammes per square centimetre, and the molecules are $3{\cdot}0 \times 10^{-8}$ cm. in diameter, then the bond per pair is about 10^{-8} dynes.

Similarly, it will be found that if the energy of a gas molecule is converted to potential during a travel through half the mean free path, the mean resistance is about 10^{-8} dynes.

It appears, then, that the molecular bond at the condition of rupture is from 10^{-8} to 10^{-6} dynes. The attraction is therefore slightly more than this, and at the point of equilibrium the repulsion must also be more than this, since it balances the attraction, and both increase with proximity.

Tentative Formula for the Attraction.

Pending a better rule, the following formula is proposed for the attraction:—

$$t_2 = \frac{Gm^2}{d^{\left(2+\frac{4d_0}{d}\right)}},$$

where G is the Newtonian constant of gravitation,
m is the mass of a molecule,
d is the molecular interval (centre to centre),
d_0 is the molecular diameter.

At absolute zero, $d=d_0$ and the exponent becomes 6, and $t_2=Gs^2$, where s is the density.

For iron this makes the gross attraction $3{\cdot}9 \times 10^{-6}$ dynes

when the molecules are in contact, $4{\cdot}4\times10^{-9}$ when the extension is 10 per cent., and becomes almost identical with gravity when the extension is 1,000 per cent. The ratio to Newtonian force falls from $1{\cdot}68\times10^{-3}$ to unity as the extension proceeds.

For ice the contact bond is $7{\cdot}2\times10^{-8}$ dynes, falling to $4{\cdot}1\times10^{-13}$ for a 20 per cent. extension. The contact condition gives a ratio to Newtonian force of $1{\cdot}2\times10^{29}$.

This formula thus gives a bond descending from about one-hundredth of the electrostatic force to the ordinary Newtonian values as the proximity decreases from absolute contact to some ten molecular diameters.

The appropriate repulsion, which is presumably largely due to kinetic energy, must decrease with great rapidity as the molecules are separated. It will differ for different molecules, and for iron a form

$$t_1=Kd^{-\gamma_1},$$

where K is about 10^{-500} and γ_1 is about 70, seems to apply.

As collateral evidence should perhaps be included the observed facts that current gradually produces brittleness in metal wires, and that cohesive strength is slightly decreased during conduction. An electrified liquid evaporates more rapidly than a neutral one, and most substances expand slightly when charged. Further than this, it may be suggested that the production of frictional electricity is due to the disturbance or rupture of cohesive fields.

REFERENCES.

A. "The Cohesion of Solids," "Proc. Phys. Soc.," Vol. XXVII., Part V., Aug. 15, 1915.

B. "Cohesion" (Second Paper), Vol. XXVIII., Part V., Aug. 15, 1916.

ABSTRACT.

(*a*) To re-state and add further evidence in favour of an electrical theory of cohesion.

(*b*) To provide tentative empirical formulæ for the expression of inter-molecular forces.

The author defines cohesion as the net attraction (*i.e.*, balance of attraction over repulsion) between molecules which are relatively chemically saturated, at distances not greatly exceeding the molecular diameters, and the following formula is proposed for this attraction :

$$t_2=Gm^2/a^{(2+4d_0/d)},$$

where G is the Newtonian constant of gravitation, m the molecular

mass, d the molecular interval (centre to centre), and d_0 is the molecular diameter.

DISCUSSION.

Dr. R. S. WILLOWS (communicated remarks) pointed out that the statement that an electrified liquid evaporated more quickly than a neutral one is not true in all cases. C. T. R. Wilson's fog experiments show, for small drops, that evaporation is most rapid when the drops do not contain an electric ion.

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